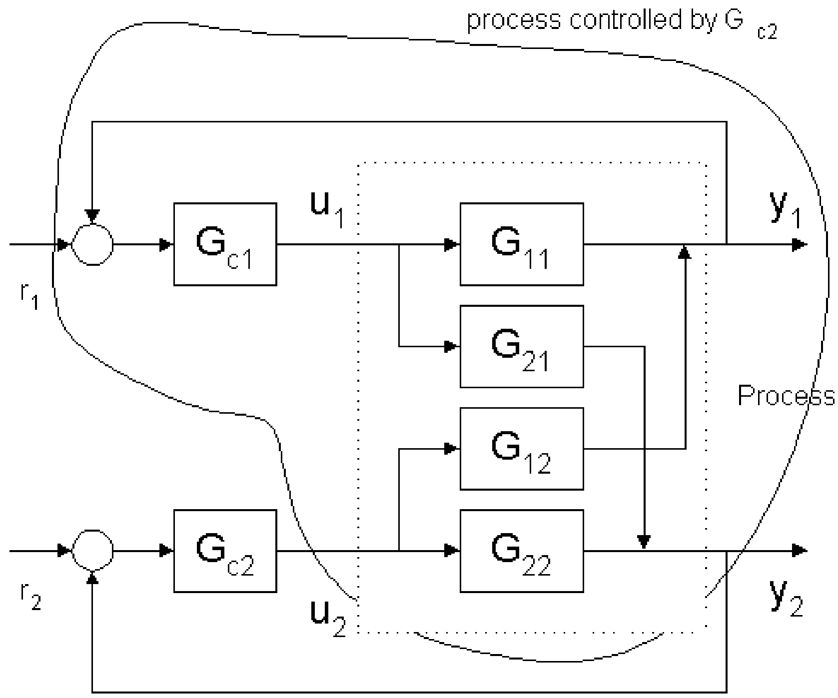
3.2 INTERACTION AND I/O PAIRING

3.2.1 INTERACTION



- When G_{c1} is open, the process that G_{c2} controls is G_{22}.
- When G_{c1} is closed, however, the process controlled by G_{c2} becomes

$$\bar{G}_{22} = G_{22} - \frac{G_{21}G_{c1}G_{12}}{1 + G_{c1}G_{11}}$$

  If both G_{12} and G_{21} are not zero, $\bar{G}_{22}$ varies with G_{c1}.

  If G_{c1} is adjusted, G_{c2} should be retuned, too.

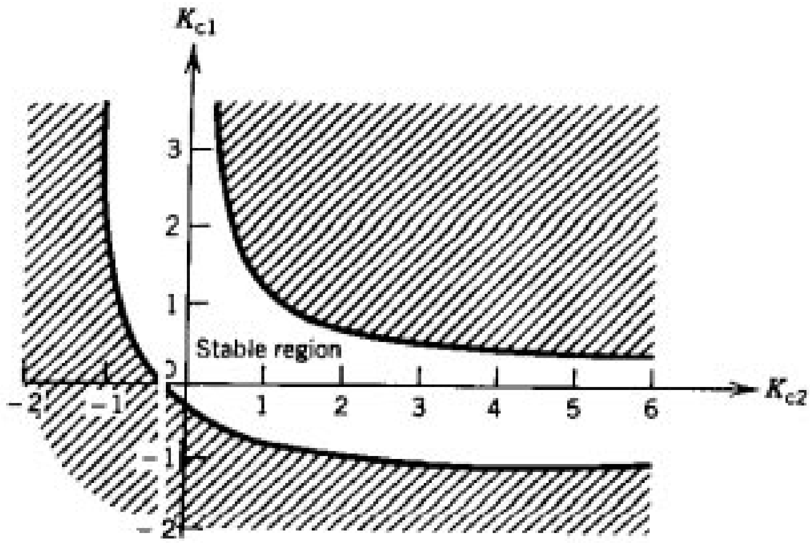
- Same thing can be said for G_{c1}.
- The above probelm is caused by the *interaction* through G_{21} and G_{12}.

Ex. Consider the following process:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{10s+1} & \frac{1.5}{s+1} \\ \frac{1.5}{s+1} & \frac{2}{10s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Assume that P-control is used for both G_{c1} and G_{c2}.

- When only one of y_1 and y_2 is under control, the controller gain can have value (as far as it is positive) without causing stability problem.
- When both control loops are closed, stability is attained for the controller gains in a limited region shown below. Both gains cannot be increased simultaneously.

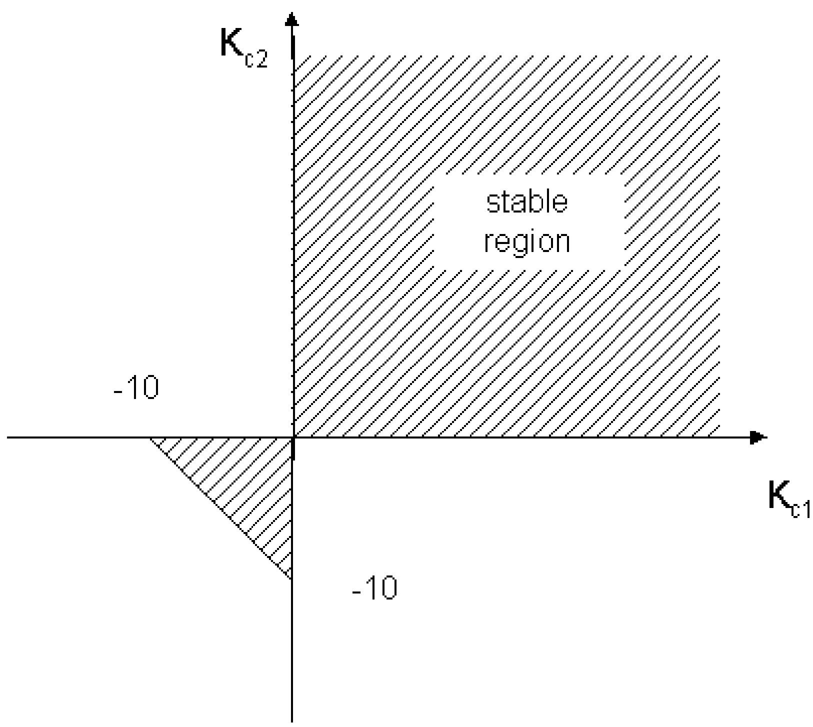


Ex. This time, we consider the process

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{10s+1} & \frac{1.5}{s+1} \\ -\frac{1.5}{s+1} & \frac{2}{10s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Again, it is assumed that P-control is used for both G_{c1} and G_{c2}.

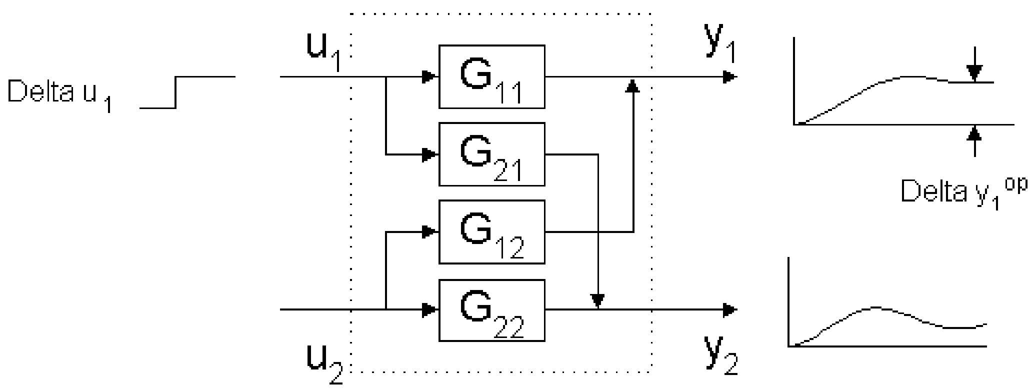
- The stability region is changed to

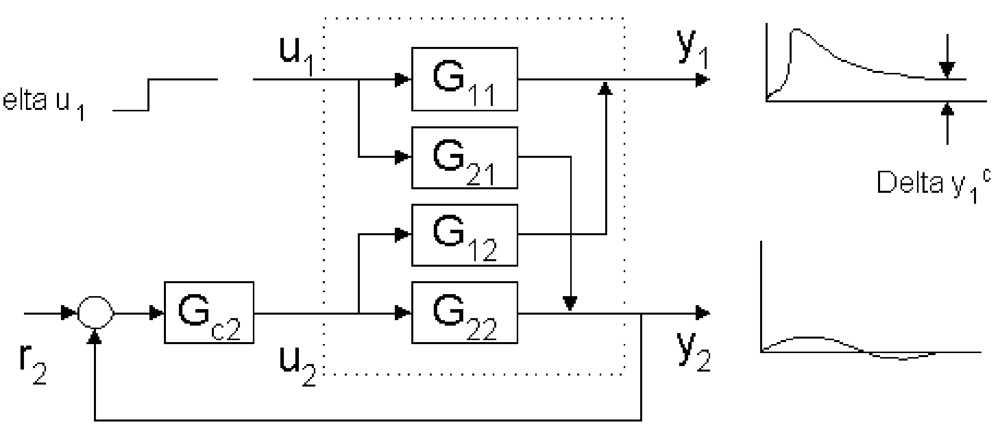


3.2.2 I/O PAIRING

- Suppose that we want to control a MIMO process using single-loop controllers (in a decentralized fashion).
- Then the pairing will be the primary question. $u_1 - y_1/u_2 - y_2$ or $u_1 - y_2/u_2 - y_1$?
- From the previous considerations, the judgement for proper pairing can be made based on an interaction measure.
- How to measure interaction ? Many different ways, but the most widely used one is the *relative gain* by Bristol(1968).

Relative Gain





Definition :

$$\begin{aligned}\lambda_{11} &= \frac{\Delta y_1^{op}/\Delta u_1}{\Delta y_1^{cl}/\Delta u_1} \\ &= \frac{\text{gain between } u_1 \text{ and } y_1 \text{ when all other loops are open}}{\text{gain between } u_1 \text{ and } y_1 \text{ when all other loops are closed}}\end{aligned}$$

- The relative gain is usually defined under steady state conditions.
- $\Lambda = [\lambda_{ij}]$ is called the *Relative Gain Array.*

Interpretations:

- Obviously, $\lambda_{11} = 1$ when G_{12} and/or G_{21} is zero. $\Rightarrow$ No interaction, $u_1 - y_1$ pair is decoupled from other loops.
- $\lambda_{11} = 0$ when $G_{11} = 0$ $\Rightarrow$ No coupling between u_1 and y_1; y_1 should be paired with u_2 for a 2×2 process.
- $\lambda_{11} > 1$ $\Rightarrow$ The gain is increased when other loops are closed.
- $\lambda_{11} < 1$ $\Rightarrow$ Sign of the gain is reversed when other loops are closed.
- $\lambda_{11} \gg 1$ or $\lambda_{11} \ll 1$ implies that the system has serious interaction. $\rightarrow$ SISO pairing has limitations. $\rightarrow$ should rely on MIMO control.

Properties:

- For an $n \times n$ process, $\sum_i \lambda_{ij} = \sum_j \lambda_{ij} = 1$.
  For a 2×2 process,

  $$\lambda_{11} = \frac{1}{1 - K_{12}K_{21}/K_{11}K_{22}}, \quad \lambda_{22} = \lambda_{11}, \quad \lambda_{12} = \lambda_{21} = 1 - \lambda_{11}$$

- The relative gain can be directly computed from the steady state gain matrix of the process.

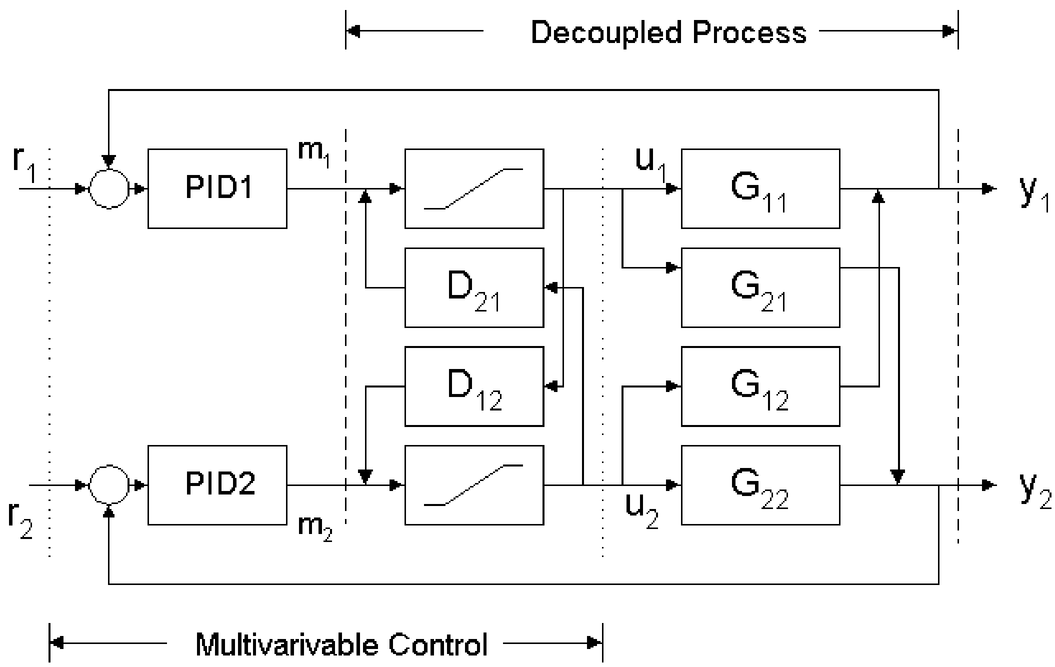
  Let $\mathbf{K}$ be the steady state gain matrix of a process.
  Then,

  $$\Lambda = \mathbf{K} \bigotimes \mathbf{K}^{-T}$$

  where $\otimes$ denotes the element-by-element multiplication.

3.3 DECOUPLING

- When λ_{11} is far from one for a 2×2 process, decentralized control has limitations. $\Rightarrow$ Multivariable Control
- One of the classical multivariable control techniques is *decoupling control.*



- **Decoupled Process**

  If we neglect the input saturation blocks,

$$
\begin{aligned}
y_1 &= (G_{11} + G_{12}D_{12})m_1 + (G_{12} + G_{11}D_{21})m_2 \\
y_2 &= (G_{21} + G_{22}D_{12})m_1 + (G_{22} + G_{21}D_{21})m_2
\end{aligned}
$$

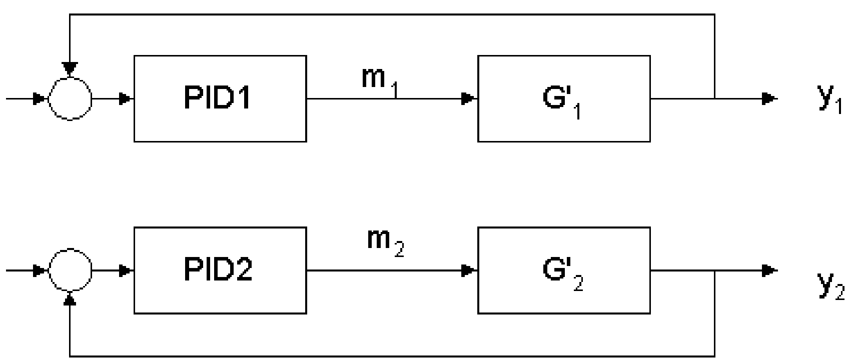
Let

$$\begin{aligned} D_{21} &= -\frac{G_{12}}{G_{11}} \\ D_{12} &= -\frac{G_{21}}{G_{22}} \end{aligned}$$

Then

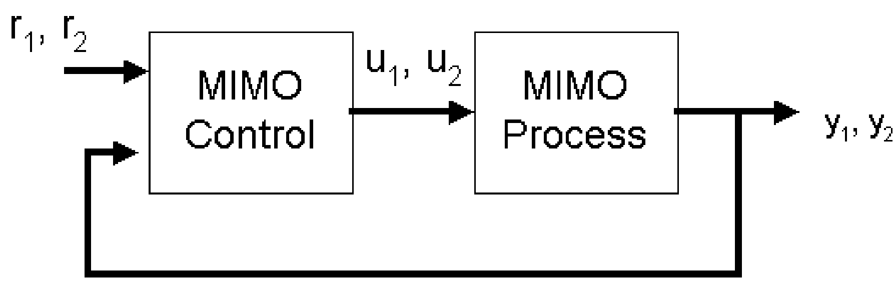
$$\begin{aligned} y_1 &= \left(G_{11} - \frac{G_{12}G_{21}}{G_{22}}\right) m_1 = G_1' m_1 \\ y_2 &= \left(G_{22} - \frac{G_{21}G_{12}}{G_{11}}\right) m_2 = G_2' m_2 \end{aligned}$$

The relative gain for the decoupled process, λ_{11d}, is one.



- **Multivariable Control**

$$\begin{aligned} u_1 &= m_1 + D_{21} m_2 \\ u_2 &= m_2 + D_{12} m_1 \end{aligned}$$

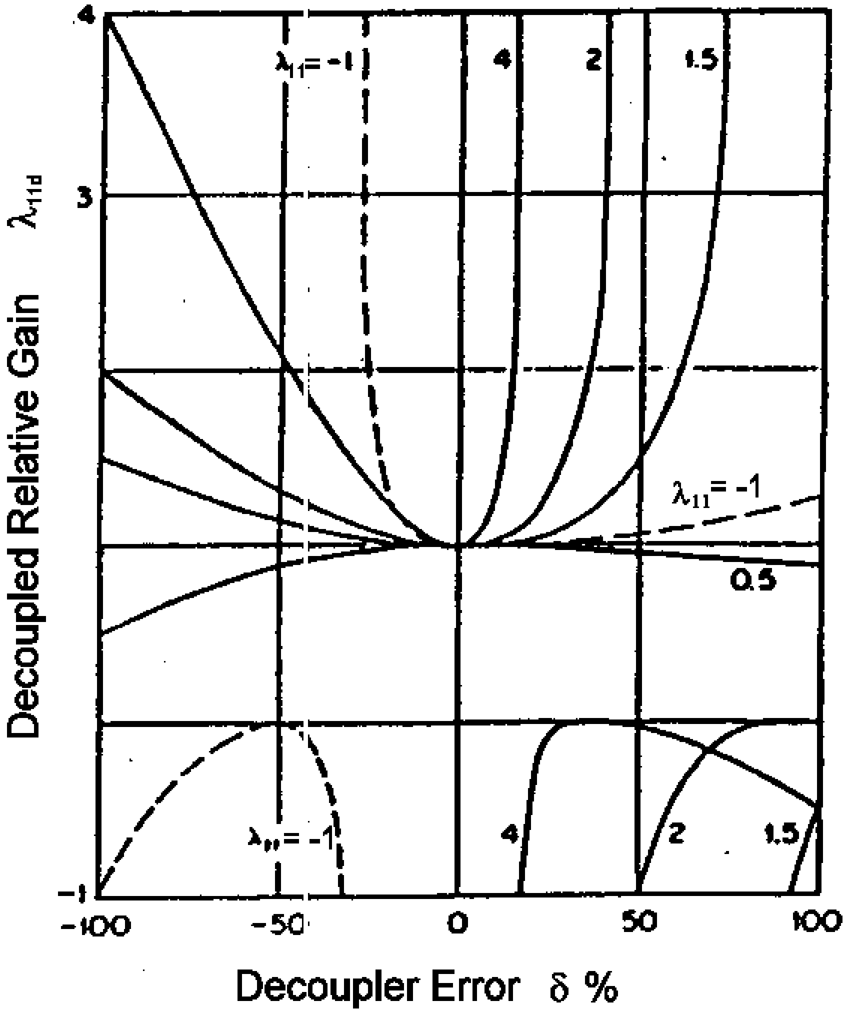


Remarks :

- A decisive drawback of decoupling control is that it is sensitive to model error.
  - For processes with $\lambda_{11} \approx 1$, $\lambda_{11d} \approx 1$.
  - As λ_{11} deviates from one, λ_{11d} also deviates from one.

  Let

$$\begin{aligned} D_{21} &= (1+\delta)\left(-\frac{G_{12}}{G_{11}}\right) \\ D_{12} &= (1+\delta)\left(-\frac{G_{21}}{G_{22}}\right) \end{aligned}$$



- The decoupler can be designed at the output.
- To design decoupling control, *Process Model* is required ! Why not try other MIMO control techniques ?