2. Estimate the area A_0.

3. Let $\tau + d = A_0/K_p$ and estimate the area A_1.

4. Then $\tau = 2.782 A_1/K_p$ and $d = A_0/K_p - \tau$

step 4 Once a FOPDT model is obtained, PID setting can be done based on a tuning rule in the next subsection.

Popular tuning rules are Quater-decay ratio setting and Integral error criterion-based setting.

1.5.3 FOPDT-BASED TUNING RULES

- The following PID tuning rules are applicable for FOPDT processes with $0.1 < d/\tau < 1$.

1/4 Decay Ratio Settings

- Z-N tuning for the FOPDT model.

Controller	K_c	T_I	T_D
P	$(\tau/K_p d)$	$--$	$--$
PI	$0.9(\tau/K_p d)$	$3.33d$	$--$
PID	$1.2(\tau/K_p d)$	$2.0d$	$0.5d$

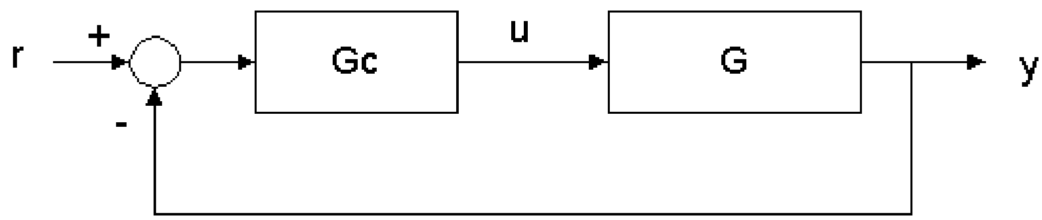
Integral Error Criteria-based Settings

- PID parameters which minimizes one of the following error-integration criteria:

$$\text{IAE} = \int_0^\infty |e(t)| dt$$

$$\text{ISE} = \int_0^\infty e^2(t) dt$$

$$\text{ITAE} = \int_0^\infty t|e(t)| dt$$



IAE Tuning Relations

Type of Input	Controller	Mode	A	B
Load	PI	P	0.984	-0.986
		I	0.608	-0.707
Load	PID	P	1.435	-0.921
		I	0.878	-0.749
		D	0.482	1.137
Set Point	PI	P	0.758	-0.861
		I	1.02^b	-0.323
Set Point	PID	P	1.086	-0.869
		I	0.740^b	-0.130^b
		D	0.348	0.914

ISE Tuning Relations

Type of Input	Controller	Mode	A	B
Load	PI	P	1.305	-0.959
		I	0.492	-0.739
Load	PID	P	1.495	-0.945
		I	1.101	-0.771
		D	0.56	1.006
Set Point	PI	P	-	-
		I	-	-
Set Point	PID	P	-	-
		I	-	-
		D	-	-

ITAE Tuning Relations

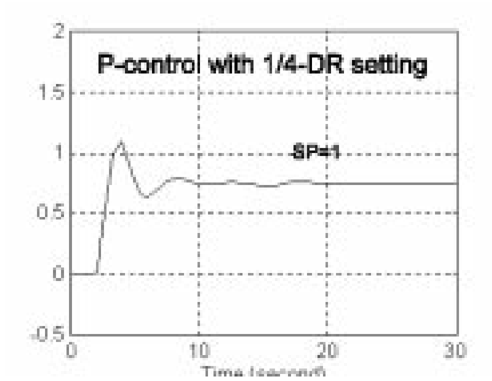
Type of Input	Controller	Mode	A	B
Load	PI	P	0.859	-0.977
		I	0.674	-0.680
Load	PID	P	1.357	-0.947
		I	0.842	-0.738
		D	0.381	0.995
Set Point	PI	P	0.586	-0.916
		I	1.03^b	-0.165
Set Point	PID	P	0.965	-0.855
		I	0.796^b	-0.147^b
		D	0.308	0.929

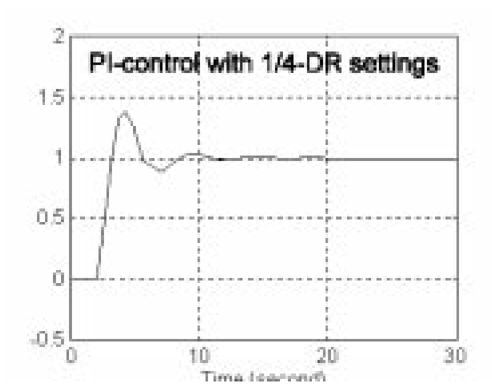
Design relation: $Y = A(d/\tau)^B$ where $Y = KK_c$ for P-mode, τ/τ_I for I-mode, and τ_D/τ for D-mode.

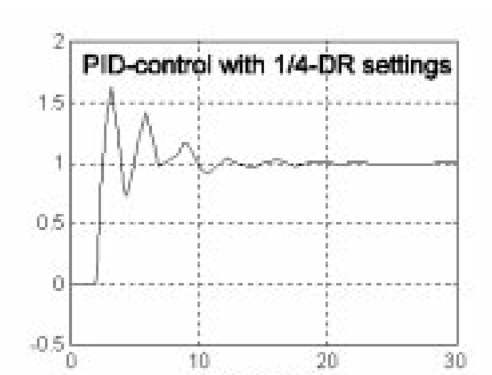
[b] For set-point change, the design relation for I-mode is $\tau/\tau_I = A + B(d/\tau)$.

Performance of 1/4-Decay Ratio Tuning

$$G(s) = \frac{e^{-s}}{3s+1}$$



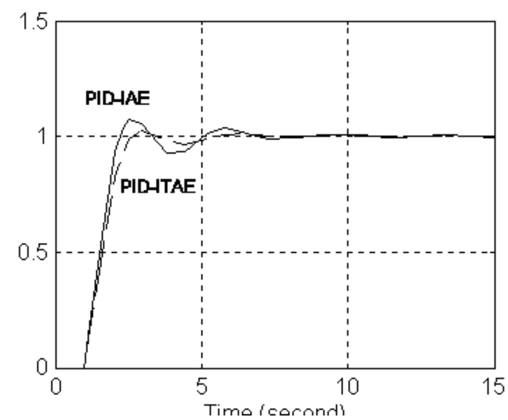




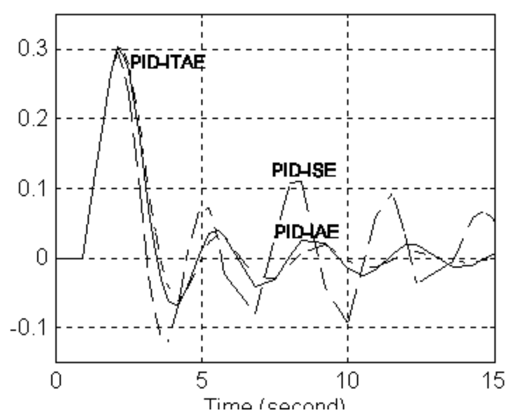
Performance of Integral Error Criteria-based Tuning

$$G(s) = \frac{e^{-s}}{3s+1}$$

set-point change



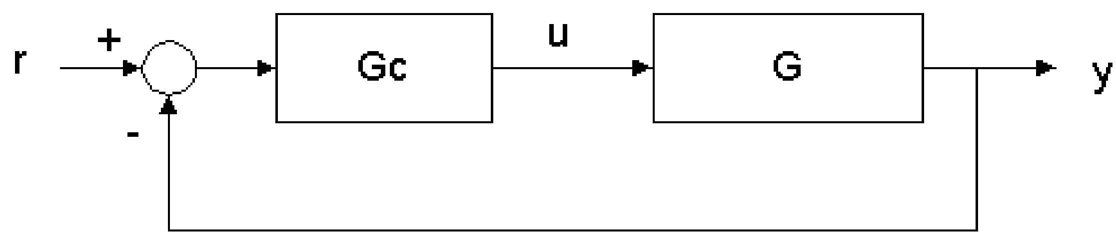
load change



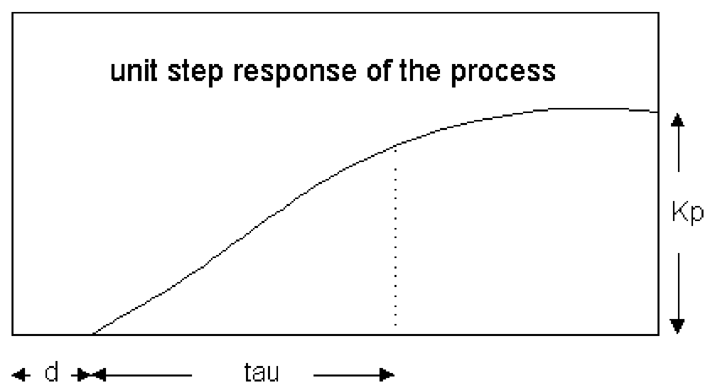
1.5.4 DIRECT SYNTHESIS METHOD - IMC TUNING

- In the direct synthesis method, a desired closed-loop response is specified first for a given process model, and then the controller which satisfies the specification is determined.
- IMC(internal model control) deals with virtually the same problem but from somewhat different point of view.

Consider a closed-loop system.

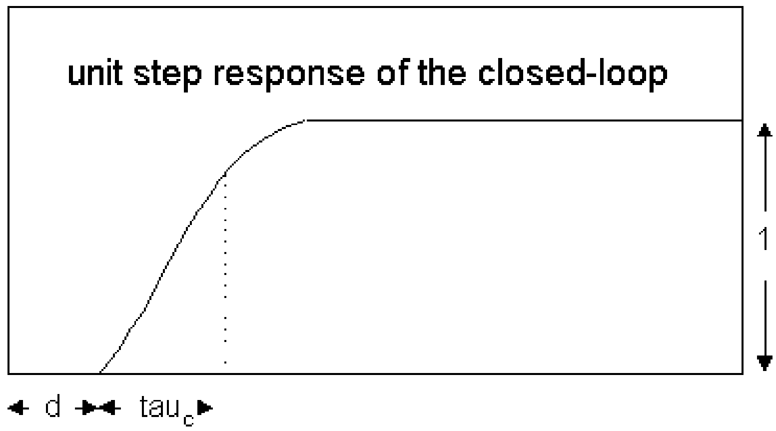


Let the unit step response of the process look like



$$y(s) = G(s)u(s) = \frac{K_p e^{-ds}}{\tau s + 1} u(s)$$

We want to design a controller which gives the following response to unit step change in set point.



$$y(s) = G_{cl}(s)r(s) = \frac{e^{-ds}}{\tau_c s + 1} r(s)$$

- Since the process has time delay of d(min), the closed-loop response should have at least d(min) of time delay.
- For zero offset error, the steady state gain of the closed-loop system is specified as unity.

G_c which satisfies the above objective ?

- From the block diagram, we have

$$y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} r(s) = G_{cl}(s)r(s) \qquad (*)$$

Solving for G_c gives

$$G_c(s) = \frac{1}{G(s)}\frac{G_{cl}(s)}{1 - G_{cl}(s)} = \frac{(\tau s + 1)/K_p}{\tau_c s - e^{-ds} + 1} \qquad --- \qquad (**)$$

When d is small,

$$e^{-ds} \approx 1 - ds$$

Therefore,

$$G_c(s) \approx \frac{(\tau s + 1)/K_p}{(\tau_c s + d)s} = \left(\frac{\tau/K_p}{\tau_c + d}\right)\left\{1 + \frac{1}{\tau s}\right\} \qquad -- \quad \text{PI-type}$$

Remarks :

- Simple and easy, but requires a model (of fairly good quality).
- τ_c is *de facto* a tuning parameter. As τ_c gets small, faster speed closed-loop response (to set point change) but more sensitive to measurement noise as well as model error. Note that tuning is reduced to a *single* parameter. Hence it is somewhere between random tuning (two or three parameters) and using PID tuning table (no parameter).
- Using (*), we can design $G_c(s)$ for arbitrary $G(s)$ and $G_{cl}(s)$ (but some restriction applies on the choice of G_{cl}). In this case, a more general type of controller other than PID is obtained. For example, (**) is in fact a Smith predictor equation.
- The following controller settings was derived for different process models based on the IMC-tuining rule.

IMC-based PID Controller Settings

Type of Model	$K_c K_p$	τ_I	τ_D
$\frac{K_p}{\tau s+1}$	$\frac{\tau}{\tau_c}$	τ	
$\frac{K_p}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\frac{\tau_1 \tau_2}{\tau_1+\tau_2}$
$\frac{K_p}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
$\frac{K_p(-\beta s+1)}{\tau^2 s^2+2\zeta\tau+1}, \beta > 1$	$\frac{2\zeta\tau}{\tau_c+\beta}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
$\frac{K_p}{s}$	$\frac{1}{\tau_c}$		
$\frac{K_p}{s(\tau s+1)}$	$\frac{1}{\tau_c}$		τ