Chapter 9: Dislocations & Strengthening Mechanisms

ISSUES TO ADDRESS...

• Why are the number of dislocations present greatest in metals?

• How are strength and dislocation motion related?

• Why does heating alter strength and other properties?
Dislocations & Materials Classes

- **Metals (Cu, Al):**
  - Dislocation motion easiest
  - Non-directional bonding
  - Close-packed directions for slip

- **Ionic Ceramics (NaCl):**
  - Motion difficult
  - Need to avoid nearest neighbors of like sign (- and +)
Dislocation Motion

Dislocation motion & plastic deformation

- Metals - plastic deformation occurs by slip – an edge dislocation (extra half-plane of atoms) slides over adjacent plane half-planes of atoms.

- If dislocations can't move, plastic deformation doesn't occur!

Fig. 9.1, *Callister & Rethwisch 9e*. (Adapted from A. G. Guy, Essentials of Materials Science, McGraw-Hill Book Company, New York, 1976, p. 153.)
Motion of Edge Dislocation

• Dislocation motion requires the successive bumping of a half plane of atoms (from left to right here).
• Bonds across the slipping planes are broken and remade in succession.

Atomic view of edge dislocation motion from left to right as a crystal is sheared.

(Courtesy P.M. Anderson)
Dislocation Motion

• A dislocation moves along a **slip plane** in a **slip direction** perpendicular to the dislocation line
• The slip direction is the same as the **Burgers vector** direction

**Fig. 9.2, Callister & Rethwisch 9e.** (Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, The Structure and Properties of Materials, Vol. III, Mechanical Behavior, p. 70. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)
Characteristics of Dislocation


Deformation Mechanisms

Slip System

- **Slip plane** - plane on which easiest slippage occurs
  - Highest planar densities (and large interplanar spacings)
- **Slip directions** - directions of movement
  - Highest linear densities

- FCC Slip occurs on \{111\} planes (close-packed) in \langle110\> directions (close-packed)
  => total of 12 slip systems in FCC
- For BCC & HCP there are other slip systems.
Burger’s vector, $b$

$b\ (FCC) = \frac{a}{2}\langle110\rangle$

$\{111\}$ planes in $\langle110\rangle$

$b\ (BCC) = \frac{a}{2}\langle111\rangle$

$\{110\}$ planes in $\langle111\rangle$

$b\ (HCP) = \frac{a}{2}\langle11\bar{2}0\rangle$

$\{0001\}$ planes in $\langle11\bar{2}0\rangle$
Stress and Dislocation Motion

- **Resolved shear stress**, $\tau_R$  
  - results from applied tensile stresses

  Applied tensile stress: $\sigma = F/A$
  Resolved shear stress: $\tau_R = F_S / A_S$

  Relation between $\sigma$ and $\tau_R$

  $$\tau_R = \frac{F_S}{A_S}$$

  $$\tau_R = \sigma \cos \lambda \cos \phi$$
Critical Resolved Shear Stress

• Condition for dislocation motion:

• Ease of dislocation motion depends on crystallographic orientation

\[ \tau_R = \sigma \cos \lambda \cos \phi \]

\(\tau_R > \tau_{\text{CRSS}}\) typically

10^{-4} \text{ GPa} \text{ to } 10^{-2} \text{ GPa}

\[ \tau = \frac{\sigma}{2} \quad \lambda = 45^\circ \quad \phi = 45^\circ \]

\[ \tau = 0 \quad \lambda = 90^\circ \]

\[ \tau = 0 \quad \phi = 90^\circ \]

\(\tau\) maximum at \(\lambda = \phi = 45^\circ\)
Single Crystal Slip

Fig. 9.8, Callister & Rethwisch 9e.

Fig. 9.9, *Callister & Rethwisch 9e.*
Ex: Deformation of single crystal

a) Will the single crystal yield?
b) If not, what stress is needed?

\[ \tau_{crss} = 20.7 \text{ MPa} \]

\[ \tau = \sigma \cos \lambda \cos \phi \]

\[ \sigma = 45 \text{ MPa} \]

\[ \tau = (45 \text{ MPa}) (\cos 35^\circ)(\cos 60^\circ) \]

\[ = (45 \text{ MPa}) (0.41) \]

\[ \tau = 18.4 \text{ MPa} \]

\[ \tau < \tau_{crss} = 20.7 \text{ MPa} \]

So the applied stress of 45 MPa will not cause the crystal to yield.
Ex: Deformation of single crystal

What stress is necessary (i.e., what is the yield stress, \( \sigma_y \))?

\[
\tau_{crss} = 20.7 \text{ MPa} = \sigma_y \cos \lambda \cos \phi = \sigma_y (0.41)
\]

\[
\therefore \sigma_y = \frac{\tau_{crss}}{\cos \lambda \cos \phi} = \frac{20.7 \text{ MPa}}{0.41} = 50.5 \text{ MPa}
\]

So for deformation to occur the applied stress must be greater than or equal to the yield stress

\[
\sigma \geq \sigma_y = 50.5 \text{ MPa}
\]
Ex: Deformation of single crystal (BCC Iron)

Stress is applied along a [010] direction.
(a) Compute $\tau_R$ along a (110) plane and in a [111] direction when a tensile stress of 52 MPa is applied.

Remember $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$

The angle between $[u_1 v_1 w_1]$ and $[u_2 v_2 w_2]$

$$\theta = \cos^{-1} \left[ \frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}} \right]$$

For $\phi$, $[u_1 v_1 w_1] = [110]$ and $[u_2 v_2 w_2] = [010]$

$$\phi = \cos^{-1} \left[ \frac{(1)(0) + (1)(1) + (0)(0)}{\sqrt{(1)^2 + (1)^2 + (0)^2} \sqrt{(0)^2 + (1)^2 + (0)^2}}} \right]$$

$$= \cos^{-1} \left( \frac{1}{\sqrt{2}} \right) = 45^\circ$$

For $\lambda$, $[u_1 v_1 w_1] = [\overline{1}11]$ and $[u_2 v_2 w_2] = [010]$

$$\lambda = \cos^{-1} \left[ \frac{(-1)(0) + (1)(1) + (1)(0)}{\sqrt{(-1)^2 + (1)^2 + (1)^2} \sqrt{(0)^2 + (1)^2 + (0)^2}}} \right]$$

$$= \cos^{-1} \left( \frac{1}{\sqrt{3}} \right) = 54.7^\circ$$
\[ \tau_R = \sigma \cos \phi \cos \lambda = (52 \text{ MPa}) (\cos 45^\circ)(\cos 54.7^\circ) \\
= 21.3 \text{ MPa} \]

(b) If slip occurs on a (110) plane and in a [\bar{1}11] direction, and the critical resolved shear stress is 30 MPa, calculate the magnitude of the applied tensile stress necessary to initiate yielding.

The yield strength \( \sigma_y \) may be computed

\[ \sigma_y = \frac{\tau_{crss}}{\cos \lambda \cos \phi} = \frac{30 \text{ MPa}}{(\cos 45^\circ)(\cos 54.7^\circ)} \]

\[ = 73.4 \text{ MPa} \]
Slip Motion in Polycrystals

- Polycrystals stronger than single crystals – grain boundaries are barriers to dislocation motion.

- Slip planes & directions \((\lambda, \phi)\) change from one grain to another.

- \(\tau_R\) will vary from one grain to another.

- The grain with the largest \(\tau_R\) yields first.

- Other (less favorably oriented) grains yield later.

Adapted from Fig. 9.10, Callister & Rethwisch 9e.
(Photomicrograph courtesy of C. Brady, National Bureau of Standards [now the National Institute of Standards and Technology, Gaithersburg, MD].)
Anisotropy in $\sigma_y$

- Can be induced by rolling a polycrystalline metal

  - before rolling
  - after rolling

- isotropic
  - since grains are equiaxed & randomly oriented.

- anisotropic
  - since rolling affects grain orientation and shape.

Anisotropy in Deformation

1. Cylinder of tantalum machined from a rolled plate:

2. Fire cylinder at a target.

3. Deformed cylinder

• The noncircular end view shows anisotropic deformation of rolled material.

Four Strategies for Strengthening:
1: Reduce Grain Size

- Grain boundaries are barriers to slip.
- Barrier "strength" increases with increasing angle of misorientation.
- Smaller grain size: more barriers to slip.

- Hall-Petch Equation:
  \[ \sigma_{yield} = \sigma_0 + k_y d^{-1/2} \]
Four Strategies for Strengthening: 2: Form Solid Solutions

- Impurity atoms distort the lattice & generate lattice strains.
- These strains can act as barriers to dislocation motion.

- Smaller substitutional impurity
  - Impurity generates local stress at A and B that opposes dislocation motion to the right.

- Larger substitutional impurity
  - Impurity generates local stress at C and D that opposes dislocation motion to the right.
Lattice Strains Around Dislocations

Fig. 9.4, Callister & Rethwisch 9e.
Strengthening by Solid Solution Alloying

• Small impurities tend to concentrate at dislocations (regions of compressive strains) - partial cancellation of dislocation compressive strains and impurity atom tensile strains

• Reduce mobility of dislocations and increase strength

Fig. 9.17, Callister & Rethwisch 9e.
Strengthening by Solid Solution Alloying

- Large impurities tend to concentrate at dislocations (regions of tensile strains)

Fig. 9.18, Callister & Rethwisch 9e.
Ex: Solid Solution Strengthening in Copper

- Tensile strength & yield strength increase with wt% Ni.

- Empirical relation: \( \sigma_y \sim C^{1/2} \)

- Alloying increases \( \sigma_y \) and TS.
Four Strategies for Strengthening:
3: Precipitation Strengthening

• Hard precipitates are difficult to shear. Ex: Ceramics in metals (SiC in Iron or Aluminum).

Result:
\[ \sigma_y \sim \frac{1}{S} \]
Application: Precipitation Strengthening

• Internal wing structure on Boeing 767

• Aluminum is strengthened with precipitates formed by alloying.
Four Strategies for Strengthening: 4: Cold Work (Strain Hardening)

- Deformation at room temperature (for most metals).
- Common forming operations reduce the cross-sectional area:

- **Forging**
  - Adapted from Fig. 17.2, Callister & Rethwisch 9e.

- **Drawing**
  - Adapted from Fig. 17.2, Callister & Rethwisch 9e.

- **Rolling**
  - Adapted from Fig. 17.2, Callister & Rethwisch 9e.

- **Extrusion**
  - Adapted from Fig. 17.2, Callister & Rethwisch 9e.

\[%CW = \frac{A_o - A_d}{A_o} \times 100\]
Dislocation Structures Change During Cold Working

- Dislocation structure in Ti after cold working.
- Dislocations entangle with one another during cold work.
- Dislocation motion becomes more difficult.

Fig. 6.12, Callister & Rethwisch 9e.
(Courtesy of M.R. Plichta, Michigan Technological University.)
Dislocation Density Increases During Cold Working

Dislocation density = \( \frac{\text{total dislocation length}}{\text{unit volume}} \)

- Carefully grown single crystals
  \( \rightarrow \) ca. \( 10^3 \) mm\(^{-2} \)
- Deforming sample increases density
  \( \rightarrow \) \( 10^9-10^{10} \) mm\(^{-2} \)
- Heat treatment reduces density
  \( \rightarrow \) \( 10^5-10^6 \) mm\(^{-2} \)

• Yield stress increases as \( \rho_d \) increases:
Impact of Cold Work

As cold work is increased

- Yield strength ($\sigma_y$) increases.
- Tensile strength ($TS$) increases.
- Ductility ($\%EL$ or $\%AR$) decreases.

Adapted from Fig. 9.20, Callister & Rethwisch 9e.
Mechanical Property Alterations Due to Cold Working

• What are the values of yield strength, tensile strength & ductility after cold working Cu?

\[
%\text{CW} = \left( \frac{\pi D^2_o - \pi D^2_d}{4 \frac{\pi D^2_o}{4}} \right) \times 100
\]

\[
%\text{CW} = \left( \frac{D^2_o - D^2_d}{D^2_o} \right) \times 100
\]

\[
%\text{CW} = \left( \frac{(15.2 \text{ mm})^2 - (12.2 \text{ mm})^2}{(15.2 \text{ mm})^2} \right) \times 100 = 35.6\%
\]
Mechanical Property Alterations Due to Cold Working

- What are the values of yield strength, tensile strength & ductility for Cu for %CW = 35.6%?

\[
\begin{align*}
\sigma_y &= 300 \text{ MPa} \\
TS &= 340 \text{ MPa} \\
\%EL &= 7%
\end{align*}
\]

Effect of Heat Treating After Cold Working

- 1 hour treatment at $T_{\text{anneal}}$...
  decreases $TS$ and increases $\%EL$.
- Effects of cold work are nullified!

![Graph showing the effect of annealing temperature on tensile strength and ductility.]

- Three Annealing stages:
  1. Recovery
  2. Recrystallization
  3. Grain Growth

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Fig. 9.22, *Callister & Rethwisch 9e.*
Reproduced by permission of ASM International, Materials Park, OH.)
Three Stages During Heat Treatment:

1. Recovery

Reduction of dislocation density by annihilation.

- **Scenario 1**
  - Results from diffusion
  - Scenario 1
  - Recovery

- **Scenario 2**
  - Scenario 2
  - "Climbed" disl. can now move on new slip plane
  - grey atoms leave by vacancy diffusion allowing disl. to "climb"
  - 1. dislocation blocked; can’t move to the right
  - 2. grey atoms leave by vacancy diffusion allowing disl. to "climb"
  - 3. "Climbed" disl. can now move on new slip plane
  - 4. opposite dislocations meet and annihilate

Dislocations annihilate and form a perfect atomic plane.
Three Stages During Heat Treatment:
2. Recrystallization

- New grains are formed that:
  -- have low dislocation densities
  -- are small in size
  -- consume and replace parent cold-worked grains.

Adapted from Fig. 9.21 (a),(b), Callister & Rethwisch 9e.
(Photomicrographs courtesy of J.E. Burke, General Electric Company.)
As Recrystallization Continues…

- All cold-worked grains are eventually consumed/replaced.
Three Stages During Heat Treatment:
3. Grain Growth

- At longer times, average grain size increases.
  - Small grains shrink (and ultimately disappear)
  - Large grains continue to grow

0.6 mm 0.6 mm

After 8 s, 580°C
After 15 min, 580°C

- Empirical Relation:
  - \[ d^n - d_o^n = Kt \]
  - Exponent typ. \( \sim 2 \)
  - Grain diam. at time \( t \)
  - Empirical coefficient dependent on material and \( T \).
  - Elapsed time

Adapted from Fig. 11.21 (d),(e), Callister & Rethwisch 9e. (Photomicrographs courtesy of J.E. Burke, General Electric Company.)
• Empirical Relation:
  exponent typ. ~ 2
  coefficient dependent on material and $T$.
  grain diam. at time $t$.
  $d^n - d^n_0 = K t$
  elapsed time

Atomic diffusion across boundary

Direction of grain boundary motion


Ex: Grain Growth

A metal having a grain diameter of $8.2 \times 10^{-3}$ mm is heated to 500 °C for 12.5 min, the grain diameter increases to $2.7 \times 10^{-2}$ mm. Compute the grain diameter when a specimen of the original material is heated at 500 °C for 100 min. Assume the grain diameter exponent $n$ has a value of 2.

$$d^2 - d_o^2 = Kt$$

$d_o = 8.2 \times 10^{-3}$ mm, $d = 2.7 \times 10^{-2}$ mm, $t = 12.5$ min

$$K = \frac{d^2 - d_o^2}{t} = \frac{(2.7 \times 10^{-2} \text{ mm})^2 - (8.2 \times 10^{-3} \text{ mm})^2}{12.5 \text{ min}}$$

$$= 5.29 \times 10^{-5} \text{ mm}^2/\text{min}$$

$$d = \sqrt{d_o^2 + Kt}$$

$$= \sqrt{(8.2 \times 10^{-3} \text{ mm})^2 + (5.29 \times 10^{-5} \text{ mm}^2/\text{min}) (100 \text{ min})}$$

$$= 0.0732 \text{ mm}$$
Fig. 9.22, Callister & Rethwisch 9e. (Adapted from G. Sachs and K. R. Van Horn, Practical Metallurgy, Applied Metallurgy and the Industrial Processing of Ferrous and Nonferrous Metals and Alloys, 1940. Reproduced by permission of ASM International, Materials Park, OH.)
Recrystallization Temperature

\[ T_R = \text{recrystallization temperature} = \text{temperature at which recrystallization just reaches completion in 1 h.} \]

\[ 0.3T_m < T_R < 0.6T_m \]

For a specific metal/alloy, \( T_R \) depends on:

- \%CW -- \( T_R \) decreases with increasing \%CW
- Purity of metal -- \( T_R \) decreases with increasing purity
Diameter Reduction Procedure - Problem

A cylindrical rod of brass originally 10 mm in diameter is to be cold worked by drawing. The circular cross section will be maintained during deformation. A cold-worked tensile strength in excess of 380 MPa and a ductility of at least 15 %EL are desired. Furthermore, the final diameter must be 7.5 mm. Explain how this may be accomplished.
Diameter Reduction Procedure - Solution

What are the consequences of directly drawing to the final diameter?

\[ \%CW = \left( \frac{A_o - A_f}{A_o} \right) \times 100 = \left( 1 - \frac{A_f}{A_o} \right) \times 100 \]

\[ = \left( 1 - \frac{\pi D_f^2/4}{\pi D_o^2/4} \right) \times 100 = \left( 1 - \left( \frac{7.5}{10} \right)^2 \right) \times 100 = 43.8\% \]
Diameter Reduction Procedure – Solution (Cont.)

• For $\%\text{CW} = 43.8%$
  
  – $\sigma_y = 420 \text{ MPa}$
  
  – $\text{TS} = 540 \text{ MPa} > 380 \text{ MPa}$
  
  – $\%\text{EL} = 6 < 15$

• This doesn’t satisfy criteria… what other options are possible?

Fig. 9.19, Callister & Rethwisch 9e.
Diameter Reduction Procedure – Solution (cont.)

For $TS > 380$ MPa  \[\Longrightarrow\]  $> 12 \%CW$

For $\%EL > 15$  \[\Longrightarrow\]  $< 27 \%CW$

\[\therefore\]  our working range is limited to $12 < \%CW < 27$

Fig. 9.19, Callister & Rethwisch 9e.
Diameter Reduction Procedure – Solution (cont.)

Cold work, then anneal, then cold work again

• For objective we need a cold work of $12 < \%CW < 27$
  – We’ll use 20 %CW

• Diameter after first cold work stage (but before 2\textsuperscript{nd} cold work stage) is calculated as follows:

\[
\%CW = \left(1 - \frac{D_{f2}^2}{D_{02}^2}\right) \times 100 \quad \Rightarrow \quad 1 - \frac{D_{f2}^2}{D_{02}^2} = \frac{\%CW}{100}
\]

\[
\frac{D_{f2}}{D_{02}} = \left(1 - \frac{\%CW}{100}\right)^{0.5} \quad \Rightarrow \quad D_{02} = \frac{D_{f2}}{\left(1 - \frac{\%CW}{100}\right)^{0.5}} = \left(1 - \frac{\%CW}{100}\right)^{0.5}
\]

Intermediate diameter = $D_{f1} = D_{02} = 7.5 \text{ mm} / \left(1 - \frac{20}{100}\right)^{0.5} = 8.39 \text{ mm}$
Diameter Reduction Procedure – Summary

Stage 1: Cold work – reduce diameter from 10 mm to 8.39 mm

\[
\%CW_1 = \left( 1 - \left( \frac{8.39 \text{ mm}}{10 \text{ mm}} \right)^2 \right) \times 100 = 29.6
\]

Stage 2: Heat treat (allow recrystallization)

Stage 3: Cold work – reduce diameter from 8.39 mm to 7.5 mm

\[
\%CW_2 = \left( 1 - \left( \frac{7.5}{8.39} \right)^2 \right) \times 100 = 20
\]

Therefore, all criteria satisfied

\[
\Rightarrow \quad \sigma_y = 340 \text{ MPa}
\]

\[
TS = 400 \text{ MPa}
\]

\[
\%EL = 24
\]
Cold Working vs. Hot Working

• Hot working $\rightarrow$ deformation above $T_R$

• Cold working $\rightarrow$ deformation below $T_R$
Grain Size Influences Properties

- **Metals having small grains** – relatively strong and tough at low temperatures

\[ \sigma_{yield} = \sigma_0 + k_y d^{-1/2} \]

- **Metals having large grains** – good creep resistance at relatively high temperatures

\[ \dot{\varepsilon} = A \sigma^n d^{-m} \exp\left(-\frac{Q}{RT}\right) \]
Summary

• Dislocations are observed primarily in metals and alloys.
• Strength is increased by making dislocation motion difficult.
• Strength of metals may be increased by:
  -- decreasing grain size
  -- solid solution strengthening
  -- precipitate hardening
  -- cold working
• A cold-worked metal that is heat treated may experience recovery, recrystallization, and grain growth – its properties will be altered.