Chapter 8: Mechanical Properties of Metals

ISSUES TO ADDRESS...

• **Stress** and **strain**: What are they and why are they used instead of load and deformation?

• **Elastic** behavior: When loads are small, how much deformation occurs? What materials deform least?

• **Plastic** behavior: At what point does permanent deformation occur? What materials are most resistant to permanent deformation?

• **Toughness** and **ductility**: What are they and how do we measure them?
Elastic Deformation

1. Initial

2. Small load

3. Unload

Elastic means reversible!

Linear-elastic

Non-Linear-elastic
Plastic Deformation (Metals)

1. Initial

2. Small load
   - Elastic + plastic
   - Bonds stretch & planes shear

3. Unload
   - Plastic strain
   - Planes still sheared

Plastic means permanent!

Linear elastic

\[ F \]

\[ \delta_{\text{elastic + plastic}} \]

\[ \delta_{\text{plastic}} \]
Engineering Stress

- **Tensile stress, \( \sigma \):**
  \[
  \sigma = \frac{F_t}{A_o} = \frac{N}{m^2}
  \]
  original cross-sectional area before loading

- **Shear stress, \( \tau \):**
  \[
  \tau = \frac{F_s}{A_o}
  \]

\[\therefore\] Stress has units: \( N/m^2 \)
Common States of Stress

- **Simple tension**: cable
  \[ \sigma = \frac{F}{A_o} \]
  \( A_o \) = cross-sectional area (when unloaded)

- **Torsion** (a form of shear): drive shaft
  \[ \tau = \frac{F_s}{A_o} \]
  \( M \), \( 2R \)
OTHER COMMON STRESS STATES (i)

- **Simple compression:**

\[ \sigma = \frac{F}{A_o} \]

Note: compressive structure member \((\sigma < 0 \text{ here}).\)
OTHER COMMON STRESS STATES (ii)

- **Bi-axial tension:**

  \[ \sigma_\theta > 0 \]
  \[ \sigma_z > 0 \]

- **Hydrostatic compression:**

  \[ \sigma_h < 0 \]

Pressurized tank (photo courtesy P.M. Anderson)

Fish under water (photo courtesy P.M. Anderson)
Engineering Strain

- **Tensile strain:**
  \[ \varepsilon = \frac{\delta}{L_0} \]

- **Lateral strain:**
  \[ \varepsilon_L = -\frac{\delta_L}{W_0} \]

- **Shear strain:**
  \[ \gamma = \frac{\Delta x}{y} = \tan \theta \]

Strain is always dimensionless.

Adapted from Fig. 8.1 (a) and (c), *Callister & Rethwisch 9e.*
Stress-Strain Testing

- Typical tensile test machine

Fig. 8.3, Callister & Rethwisch 9e.

- Typical tensile specimen

Fig. 8.2, Callister & Rethwisch 9e.
Linear Elastic Properties

- Modulus of Elasticity, $E$: (also known as Young's modulus)
- Hooke's Law:

\[ \sigma = E \varepsilon \]
Poisson's ratio, $\nu$

- **Poisson's ratio, $\nu$:** Ratio between radial and axial strains

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$

- **Units:**
  - $E$: [GPa] or [psi]
  - $\nu$: dimensionless

- **Values for different materials:**
  - Metals: $\nu \sim 0.33$
  - Ceramics: $\nu \sim 0.25$
  - Polymers: $\nu \sim 0.40$

- **Behavior of $\nu$:**
  - $\nu > 0.50$ density increases
  - $\nu = 0.50$ no volume change
  - $\nu < 0.50$ density decreases (voids form)
Mechanical Properties

- Slope of stress strain plot (which is proportional to the elastic modulus) depends on bond strength of metal

Fig. 8.7, Callister & Rethwisch 9e.
Other Elastic Properties

- Elastic Shear modulus, $G$:
  \[ \tau = G \gamma \]

- Special relations for isotropic materials:
  \[ G = \frac{E}{2(1 + \nu)} \]
Based on data in Table B.2, Callister & Rethwisch 9e. Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.
Plastic (Permanent) Deformation
(at lower temperatures, i.e. $T < T_{\text{melt}}/3$)

• Simple tension test:

engineering stress, $\sigma$

Elastic initially

Permanent (plastic) after load is removed

Elastic + Plastic at larger stress

Engineering strain, $e$

Plastic strain

Adapted from Fig. 8.10 (a), Callister & Rethwisch 9e.
Yield Strength, \( \sigma_y \)

- Stress at which *noticeable* plastic deformation has occurred.

\[
\sigma_y = \text{yield strength}
\]

when \( \varepsilon_p = 0.002 \)

Note: for 2 inch sample

\[
\varepsilon = 0.002 = \frac{\Delta z}{z}
\]

\[
\therefore \Delta z = 0.004 \text{ in}
\]

Adapted from Fig. 8.10 (a), Callister & Rethwisch 9e.
Typical stress-strain behavior for a metal

Typical stress-strain behavior for steels
Room temperature values

Based on data in Table B.4, 
Callister & Rethwisch 9e.

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered


**Tensile Strength, TS**

- Maximum stress on engineering stress-strain curve.

![Diagram](image)

- **Metals**: occurs when noticeable necking starts.
- **Polymers**: occurs when polymer backbone chains are aligned and about to break.

Adapted from Fig. 8.11, *Callister & Rethwisch 9e.*

\[ F = \text{fracture or ultimate strength} \]

Neck – acts as stress concentrator
Tensile Strength: Comparison

Based on data in Table B4, Callister & Rethwisch 9e.

- a = annealed
- hr = hot rolled
- ag = aged
- cd = cold drawn
- cw = cold worked
- qt = quenched & tempered

AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

Room temperature values
Ductility

- Plastic tensile strain at failure:

\[ \%EL = \frac{L_f - L_o}{L_o} \times 100 \]

% elongation

- Another ductility measure:

\[ \%RA = \frac{A_o - A_f}{A_o} \times 100 \]

% reduction in area

Adapted from Fig. 8.13, Callister & Rethwisch 9e.
Toughness

• Energy to break a unit volume of material
• Approximate by the area under the stress-strain curve.

Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

Adapted from Fig. 8.13, *Callister & Rethwisch 9e.*

Engineering tensile stress, $\sigma$

Engineering tensile strain, $\varepsilon$

very small toughness (unreinforced polymers)

large toughness (metals)

small toughness (ceramics)
Resilience, $U_r$

- Ability of a material to store energy
  - Energy stored best in elastic region

\[ U_r = \int_0^\gamma \sigma \, d\varepsilon \]

If we assume a linear stress-strain curve this simplifies to

\[ U_r \approx \frac{1}{2} \sigma_y \varepsilon_y = \frac{1}{2} \sigma_y \left( \frac{\sigma_y}{E} \right) = \frac{\sigma_y^2}{2E} \]

Fig. 8.15, Callister & Rethwisch 9e.
True Stress & Strain
S.A. changes when sample stretched

True Stress ($\sigma_T$)
True stress is the stress determined by the instantaneous load acting on the instantaneous cross-sectional area

True Strain ($\varepsilon_T$)
The rate of instantaneous increase in the instantaneous gauge length.

Adapted from Fig. 8.16, Callister & Rethwisch 9e.
True Stress & Strain vs. Engineering Stress & Strain

Assuming material volume remains constant : $A_o l_o = A_i l_i$

$$
\sigma_T = \frac{F}{A_i} = \frac{F}{A_o} \frac{A_o}{A_i} = \sigma \frac{l_i}{l_o} = \sigma \frac{l_o + \delta}{l_o} = \sigma (1 + \varepsilon)
$$

$$
\varepsilon_T = \int_{l_o}^{l_i} \frac{dl}{l} = \ln \left( \frac{l_i}{l_o} \right) = \ln \left( \frac{l_o + \delta}{l_o} \right) = \ln \left( 1 + \varepsilon \right)
$$

True stress
$$
\sigma_T = \frac{F}{A_i} \quad \sigma_T = \sigma \left( 1 + \varepsilon \right)
$$

True strain
$$
\varepsilon_T = \ln \left( \frac{l_i}{l_o} \right) \quad \varepsilon_T = \ln \left( 1 + \varepsilon \right)
$$
Hardening

- An increase in $\sigma_y$ due to plastic deformation.

- Curve fit to the stress-strain response:
  \[ \sigma_T = K \left( \frac{\epsilon_T}{\epsilon_f} \right)^n \]
  
  - "true" stress ($F/A$)
  - "true" strain: $\ln(\ell/\ell_0)$
  - Hardening exponent:
    - $n = 0.15$ (some steels)
    - to $n = 0.5$ (some coppers)
Elastic Strain Recovery

1. Load
2. Unload
3. Reapply load

Fig. 8.17, Callister & Rethwisch 9e.

σ_{y_i}
σ_{y_o}
Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  - resistance to plastic deformation or cracking in compression.
  - better wear properties.

**e.g.,** 10 mm sphere

- apply known force
- measure size of indent after removing load

- Smaller indents mean larger hardness.

**Increasing hardness**
- most plastics
- brasses
- Al alloys
- easy to machine
- steels
- file hard
- cutting tools
- nitrided steels
- diamond
Hardness: Measurement

• Rockwell
  – No major sample damage
  – Each scale runs to 130 but only useful in range 20-100.
  – Minor load 10 kg
  – Major load 60 (A), 100 (B) & 150 (C) kg
    • A = diamond, B = 1/16 in. ball, C = diamond

• HB = Brinell Hardness
  – TS (MPa) = 3.45 x HB
# Hardness: Measurement

**Table 8.5  Hardness Testing Techniques**

<table>
<thead>
<tr>
<th>Test</th>
<th>Indenter</th>
<th>Shape of Indentation</th>
<th>Load</th>
<th>Formula for Hardness Number&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brinell</td>
<td>10-mm sphere of steel or tungsten carbide</td>
<td></td>
<td>$P$</td>
<td>$\text{HB} = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$</td>
</tr>
<tr>
<td>Vickers microhardness</td>
<td>Diamond pyramid</td>
<td></td>
<td>$P$</td>
<td>$\text{HV} = 1.854P/d_1^2$</td>
</tr>
<tr>
<td>Knoop microhardness</td>
<td>Diamond pyramid</td>
<td></td>
<td>$P$</td>
<td>$\text{HK} = 14.2P/l^2$</td>
</tr>
<tr>
<td>Rockwell and</td>
<td>Diamond cone $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres</td>
<td></td>
<td>60 kg, 100 kg, 150 kg, 15 kg, 30 kg, 45 kg</td>
<td></td>
</tr>
<tr>
<td>Rockwell Superficial</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

<sup>a</sup> For the hardness formulas given, $P$ (the applied load) is in kg, while $D$, $d$, $d_1$, and $l$ are all in mm.

Design or Safety Factors

• Design uncertainties mean we do not push the limit.
• Factor of safety, $N$

$$\sigma_{\text{working}} = \frac{\sigma_y}{N}$$

Often $N$ is between 1.2 and 4

• Example: Calculate a diameter, $d$, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

1045 plain carbon steel:

$$\sigma_y = 310 \text{ MPa}$$
$$TS = 565 \text{ MPa}$$

Given:

$$F = 220,000 \text{ N}$$
$$L_0 = 0.067 \text{ m} = 6.7 \text{ cm}$$

$$\frac{220,000 \text{ N}}{\pi \left( d^2 / 4 \right)} = \frac{\sigma_y}{N}$$

$$5$$

$$d = 0.067 \text{ m} = 6.7 \text{ cm}$$
Summary

• **Stress** and **strain**: These are size-independent measures of load and displacement, respectively.

• **Elastic** behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus ($E$ or $G$).

• **Plastic** behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches $\sigma_y$.

• **Toughness**: The energy needed to break a unit volume of material.

• **Ductility**: The plastic strain at failure.