# Bioheat Transfer

# introduction

1. Deriving Governing equation
2. Boundary conditions
3. Deriving The bioheat transfer equation
4. Governing Equations for heat condition in various coordinate systems
5. Problem formulation

# The Bioheat Transfer Equation for Mammalian Tissue

1. The mammalian tissue system

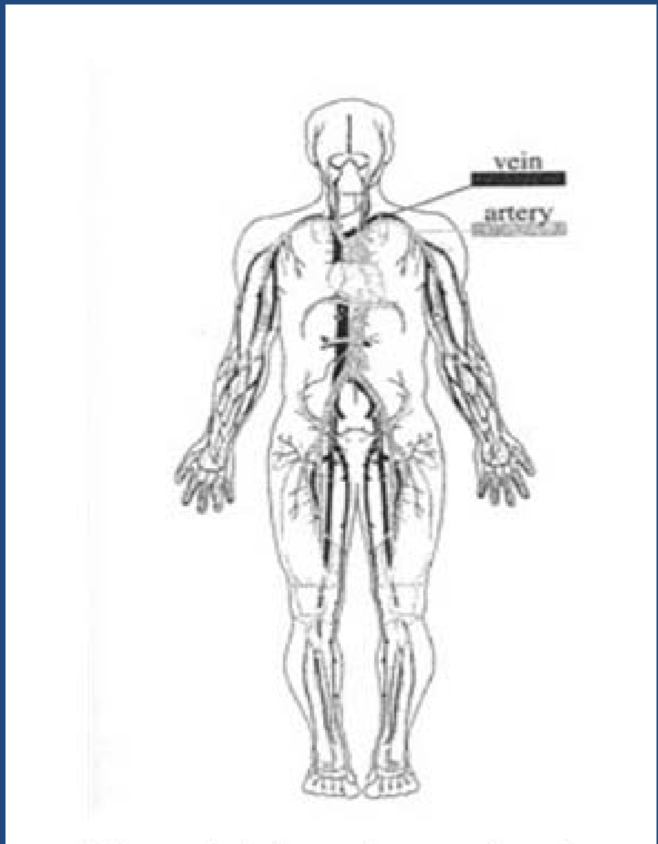



Figure 1. Arteries and veins of the circulatory system

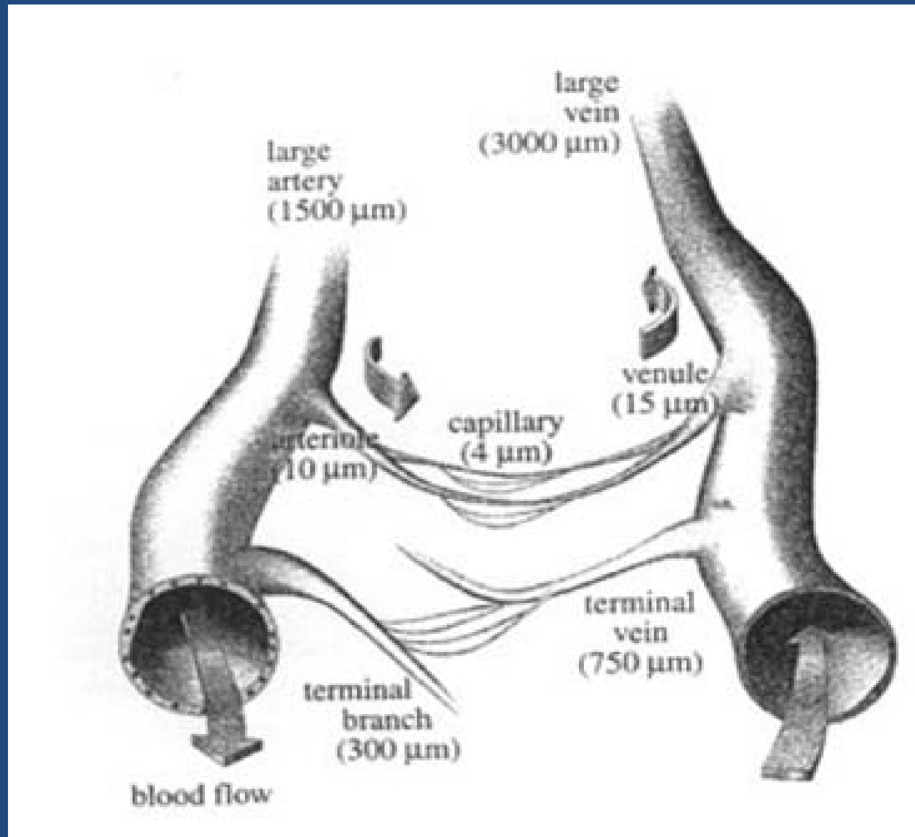



Figure 2. variation of blood vessel sizes.

# The Bioheat Transfer Equation for Mammalian Tissue

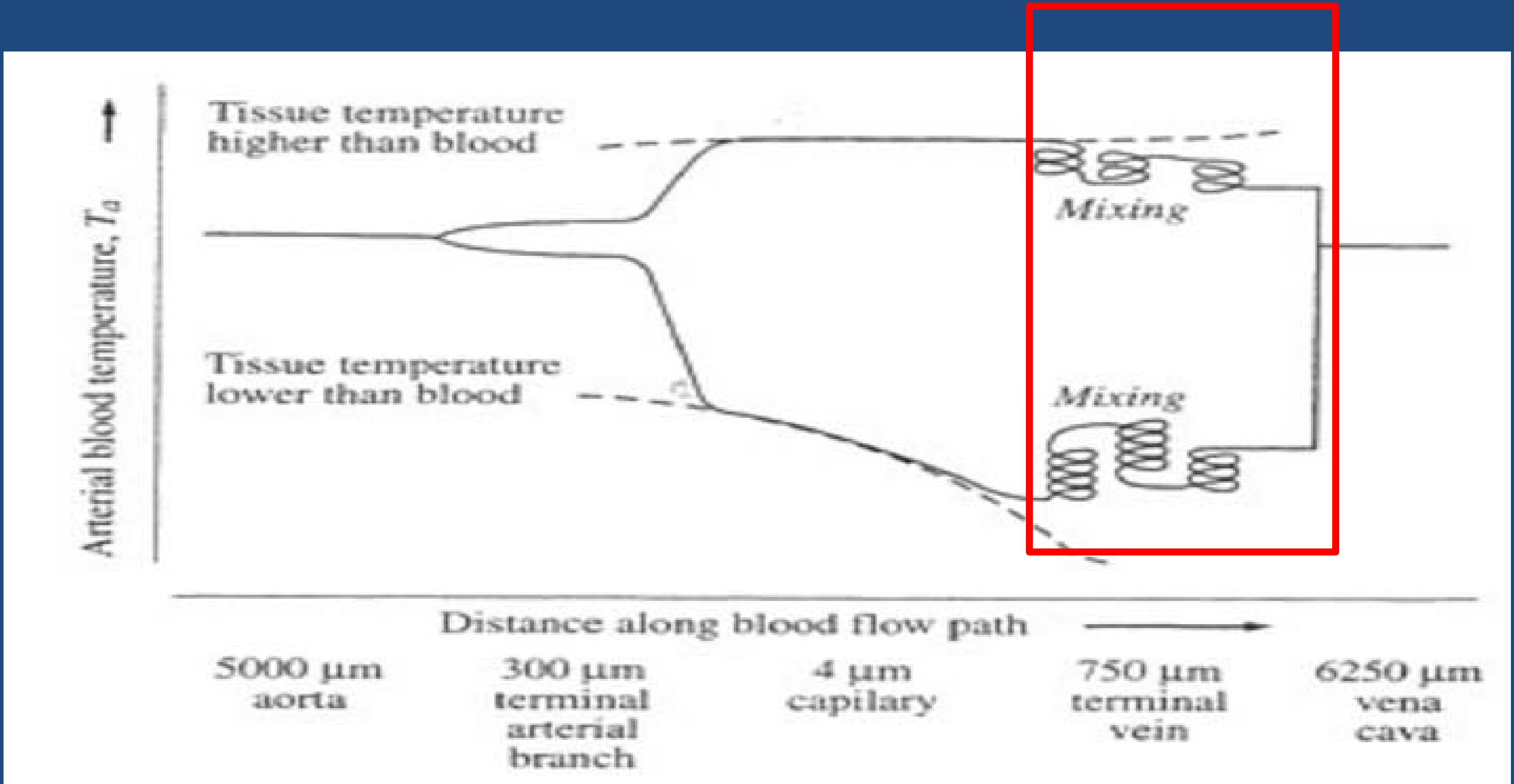



Figure 3. Variation of blood temperature in the blood vessels.

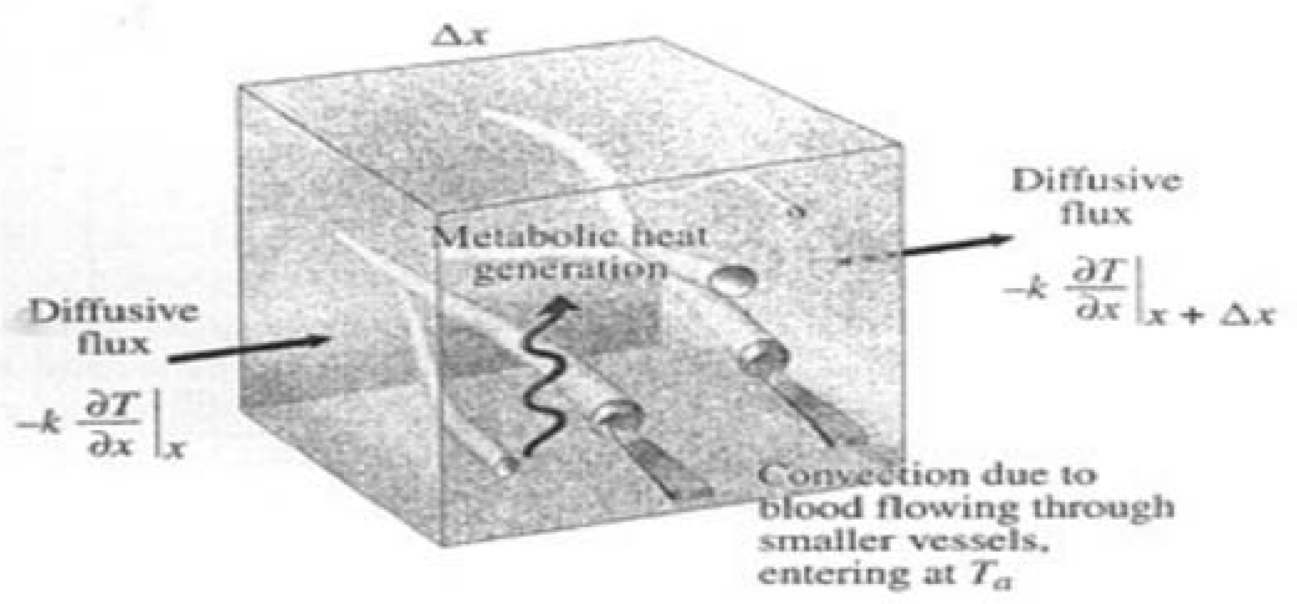



Figure 4. Idealized heat transfer in a tissue showing metabolic heat generation Q and convective heat transfer due to the passage of blood.

# The bioheat transfer equation

Assumptions

1) Homogeneous material with isotropic thermal properties
2) Large blood vessels are ignored
3) Blood capillaries are isotropic

Blood is at arterial temperature but quickly reaches the tissue temperature by the time it reaches the end of the artery system.

- The governing bioheat equation

$$\underbrace{\rho c \frac{\partial T}{\partial t}}_{\text{Change in storage}} = \underbrace{k \nabla^2 T}_{\text{conduction}} + \underbrace{\rho_b c_b V_b^v (T_a - T)}_{\text{Convection Due to blood flow}} + \underbrace{Q}_{\text{generation}}$$

Change in storage | conduction | Convection Due to blood flow | generation

# Governing Equation Derived in Cylindrical

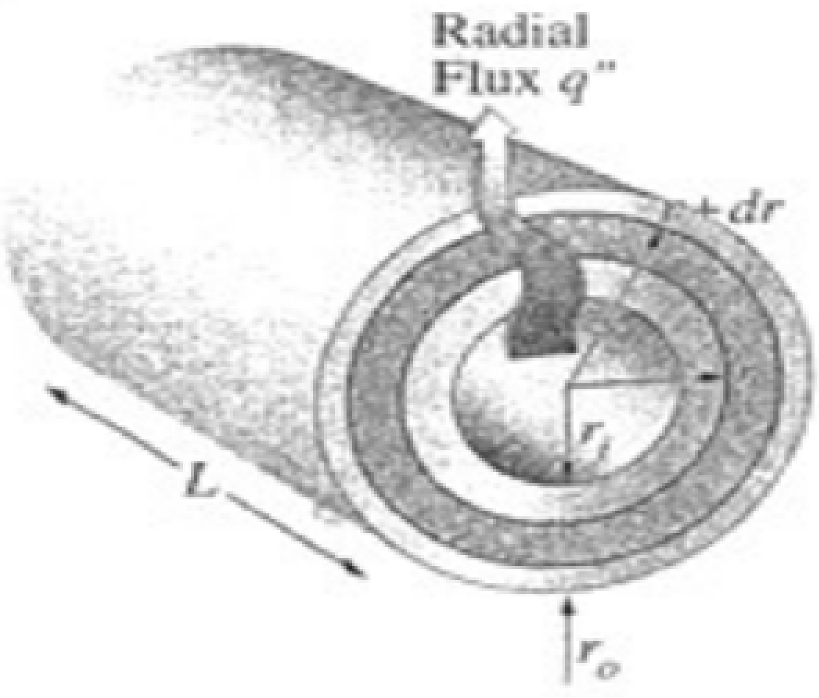



Figure 5. Radial heat flow in a long cylinder.

# 3.4 Governing Equation Derived in Cylindrical

| Energy In | - | Energy Out | + | Energy Generated | = | Energy Stored |
|---|---|---|---|---|---|---|

$$[2\pi r L q_r'' - 2\pi(r+\Delta r)L q_{r+\Delta r}'' + 2\pi r \Delta r L Q]\Delta t = 2\pi r \Delta r L \rho c_p \Delta T$$

$$-\frac{((r+\Delta r)q_{r+\Delta r}'' - r q_r'')}{r\Delta r} + Q = \rho c_p \frac{\Delta T}{\Delta t}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}(r q_r'') + Q = \rho c_p \frac{\partial T}{\partial t}$$

Using Fourier's law: $$k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + Q = \rho c_p \frac{\partial T}{\partial t}$$

Governing Equation in cylindrical:

$$\underbrace{\frac{\partial T}{\partial t}}_{\text{storage}} = \underbrace{\frac{k}{\rho c_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)}_{\text{conduction}} + \underbrace{\frac{Q}{\rho c_p}}_{\text{generation}}$$

# Governing Equation for Heat Condition in Various Coordinate Systems

- Cartesian

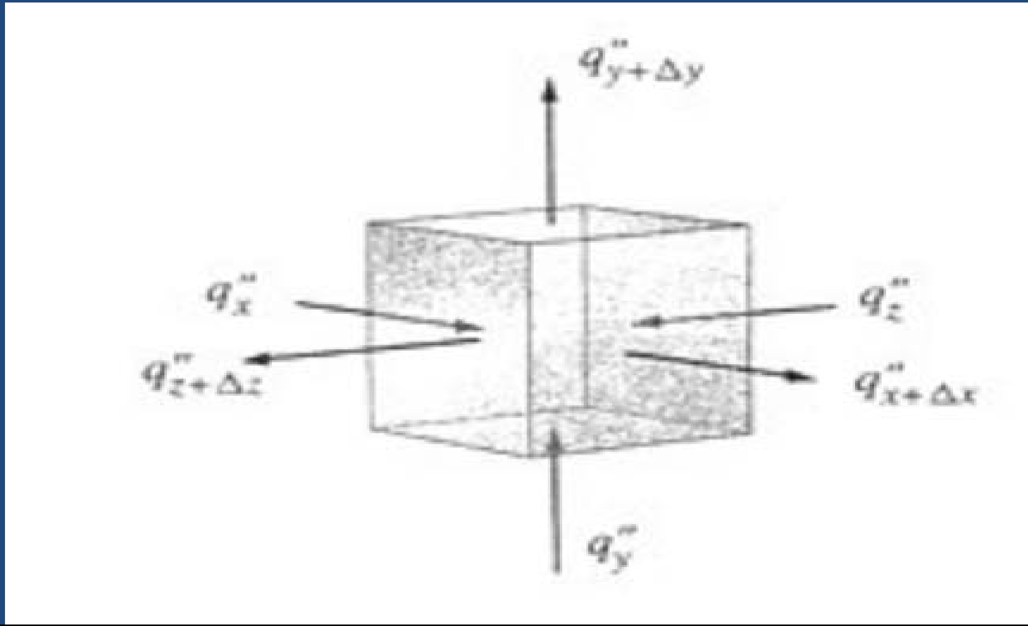



Figure 5, Energy balance over a control volume in a Cartesian coordinate system.

$$\frac{k}{\rho c_p}\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+\frac{Q}{\rho c_p}=\frac{\partial T}{\partial t}$$

# Governing Equation for Heat Condition in Various Coordinate Systems

- Cylindrical

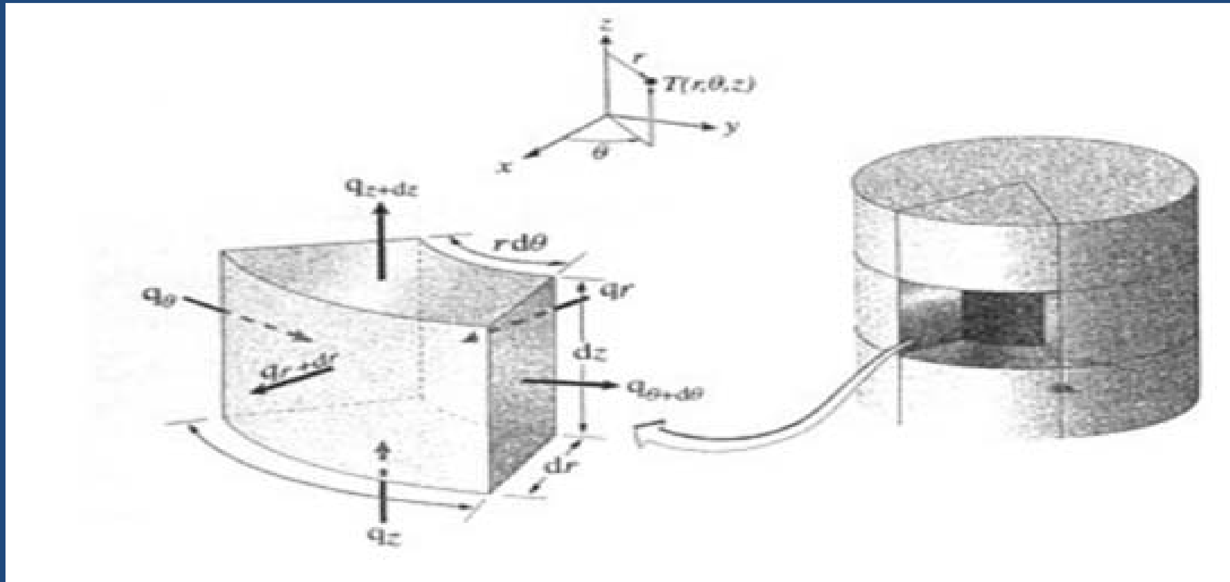



Figure 6. Energy balance over a control volume in a cylindrical coordinate system.

$$\frac{k}{\rho c_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2 T}{\partial \Phi^2}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\frac{Q}{\rho c_p}=\frac{\partial T}{\partial t}$$

# Governing Equation for Heat Condition in Various Coordinate Systems

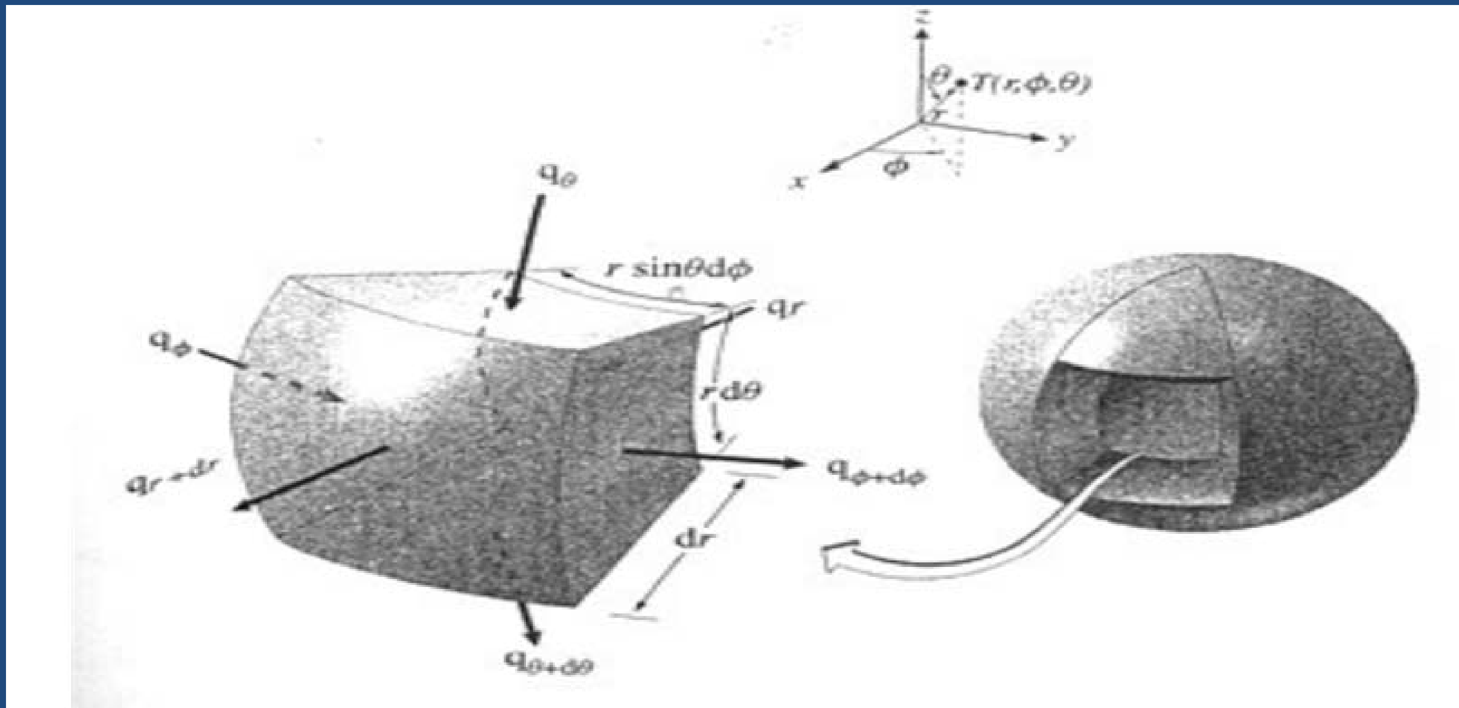
- 


Figure 7.Energy balance over a control volume in a spherical coordinate system.

$$\frac{k}{\rho c_p}\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r})+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \Phi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial T}{\partial\theta})\right]+\frac{Q}{\rho c_p}=\frac{\partial T}{\partial t}$$

# Governing Equation for Heat Condition in Various Coordinate Systems

- Symbolically (Any coordinate system)

$$\frac{k}{\rho c_p} \nabla^2 T + \frac{Q}{\rho c_p} = \frac{\partial T}{\partial t}$$

# An Algorithm to Solve Transport Problem

**Schematic of the problem**

**Governing Equation**

- Is spatial variation needed (check Bio number)
- What geometry/coordinate system?
- Which terms should be dropped?

**Boundary Conditions**

- How many conditions are necessary?
- What type of boundary conditions?
- Is an initial condition needed (not for steady state)?

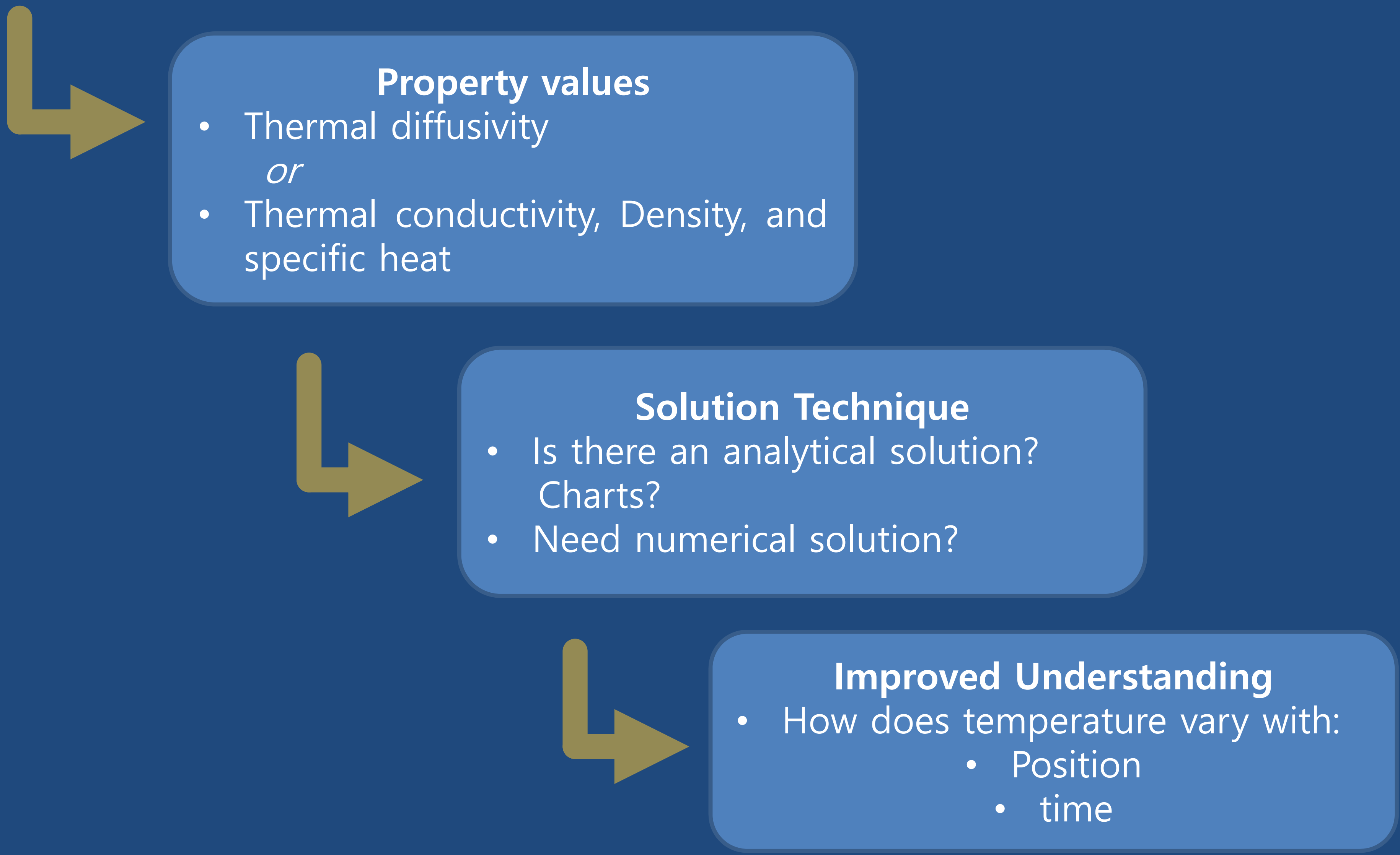



Figure 8. A step by step procedure to solve heat and mass transfer problems, showing the steps in case of a heat transfer problem

# Summary

- The governing Bioheat Transfer Equation for Mammalian Tissue

$$\underbrace{\rho c \frac{\partial T}{\partial t}}_{\text{Change in storage}} = \underbrace{k \nabla^2 T}_{\text{conduction}} + \underbrace{\rho_b c_b V_b^v (T_a - T)}_{\text{Convection Due to blood flow}} + \underbrace{Q}_{\text{generation}}$$

- Governing Equation for Heat Condition in Various Coordinate Systems
  - Cartesian $$\frac{k}{\rho c_p}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{Q}{\rho c_p} = \frac{\partial T}{\partial t}$$
  - Cylindrical $$\frac{k}{\rho c_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 T}{\partial \Phi^2}\right) + \frac{\partial^2 T}{\partial z^2}\right] + \frac{Q}{\rho c_p} = \frac{\partial T}{\partial t}$$
  - Spherical $$\frac{k}{\rho c_p}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \Phi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)\right] + \frac{Q}{\rho c_p} = \frac{\partial T}{\partial t}$$
  - Symbolically (Any coordinate system) $$\frac{k}{\rho c_p}\nabla^2 T + \frac{Q}{\rho c_p} = \frac{\partial T}{\partial t}$$

# Summary

- Problem Formulation
  1. It is the development of mathematical formulation of a physical problems, written in terms of governing equation and boundary conditions.
  2. Follow the steps as shown in Figure 8.