Chapter 4 Movement of Particles by Fluid Flow

4.1 Fluid Flow through Packed Bed of Particles (Chapter 4)

(1) Pressure Drop - Flow Relationship

1) Laminar Flow

Fluid flow through a packed bed: simulated by fluid flow through a hypothetical tubes

\[ \therefore \frac{(-p)}{H} = \frac{32U}{D^2} \]

\[ \Rightarrow \frac{(-p)}{H_e} = \frac{K_1U_i}{D_e^2} \]

Hagen-Poiseille equation

Substituting suitable relations for \( H_e \) (equivalent height) and \( D_e \) (equivalent diameter)

\[ \therefore \frac{(-p)}{H} = 180 \frac{U}{x^2} \left(1 - \frac{\varepsilon}{3}\right)^2 \]

Carman-Kozeny equation

2) General Equation for Turbulent and Laminar Flow

Ergun equation

\[ \frac{(-p)}{H} = 150 \frac{U}{x^2} \left(1 - \frac{\varepsilon}{3}\right)^2 + 1.75 \frac{U^2}{x} \left(1 - \frac{\varepsilon}{3}\right) \]

Laminar \hspace{2cm} Turbulent

Laminar flow for \( Re \ast = \frac{xU}{(1 - \varepsilon)} < 10 \)

Turbulent flow for \( Re \ast = \frac{xU}{(1 - \varepsilon)} > 2000 \)

or

\[ f \ast = \frac{150}{Re \ast} + 1.75 \]

where \( f \ast = \frac{(-p)}{H} \frac{x}{U^2} \left(1 - \frac{\varepsilon}{3}\right) \)
3) **Nonspherical Particles**

\[ x_{sv} \text{ (surface-volume diameter)} \text{ instead of } x \]

**Worked Example 4.1**

(2) **(Liquid) Filtration**

![Diagram of filtration process]

(1) **Introduction**

**Filter media**: Canvas cloth, woolen cloth, metal cloth, glass, cloth, paper, synthetic fabrics

**Filter aids**: To avoid cake plugging

- e.g. Diatomaceous silica, perlite, purified woolen cellulose, other inert porous solids
- By either adding slurry (increasing cake permeability)
  - or precoating the filter media surface

1) **Incompressible Cake**

For cake filter

*From laminar part of Ergun equation*

\[
\frac{(-p)}{H} = 150 \frac{U(1-\varepsilon)^2}{x^2} 
\]
where \( L \): cake thickness

\( x \): surface-volume diameter of particle

* For compressible filter cake,

\[
\frac{dp}{dL} = r_c \ U
\]

where \( r_c \): a function of pressure difference

By defining cake resistance \( r_c \)

\[
r_c = \frac{150(1 - \varepsilon)^2}{x^2},
\]

\[
\frac{(- p)}{H} = r_c \ U
\]

where \( U = \frac{1}{A} \frac{dV}{dt} \)

\( V \): volume of slurry fed to filter

Also defining \( \phi \) (volume formed by passage of unit volume filtrate)

\[
\phi = \frac{HA}{V},
\]

\[
\frac{dV}{dt} = \frac{A^2 (- p)}{r_c V}
\]

Including the resistance of filter medium,

since the resistances of the cake and the filter medium are in series,

\[
(- p) = (- p_m) + (- p_c)
\]

\[
\downarrow
\]

\[
\frac{1}{A} \frac{dV}{dt} r_c H_c
\]

By analogy for the filter medium

\[
(- p_m) = \frac{1}{A} \frac{dV}{dt} r_m H_m
\]

\[
\therefore (- p) = \frac{1}{A} \frac{dV}{dt} (r_m H_m + r_c H_c)
\]

Defining equivalent height of filter cake and volume of filtrate
\[ r_m H_m = r_c H_{eq} \text{ and } H_{eq} = \frac{V_{eq}}{A} \]

\[ V_{eq} = \frac{A r_m H_m}{\phi r_c} \]

where \( V_{eq} \): volume of filtrate passing to create a cake of thickness \( H_{eq} \)

\[ \therefore \frac{1}{A} \frac{dV}{dt} = \frac{(-p)A}{r_c (V + V_{eq})} \]

Constant rate filtration

\[ \frac{1}{A} \frac{dV}{dt} = \frac{(-p)A}{r_c (V + V_{eq})} = \text{constant} \]

Constant pressure filtration

Integrating

\[ \frac{t}{V} = \frac{r_e}{A^2 (-p)} \left( \frac{V}{2} + V_{eq} \right) \]

Worked Example 4.2

3) Washing the Cake

Figure 4.2

4.2 Fluidization (Chapter 5)

(1) Fundamental

* \( p \text{ vs. } U \) Figure 5.1

Minimum (incipient) fluidization, \( U_{mf} \)

From force balance

Net downward force

\[ p = (1 - \varepsilon)(\rho - \rho)H \quad (1) \]
Net upward force

\[
\frac{p}{H} = 150 \left(1 - \frac{1}{3}\right)^2 \frac{U}{x_{sv}} + 1.75 \frac{1}{3} \frac{g U^2}{x_{sv}} \quad (2)
\]

Equating (1) and (2) at \(U = U_{mf}\)

\[
Ar = 150 \left(1 - \frac{1}{3}\right) Re_{mf} + 1.75 \frac{1}{3} Re_{mf}^2
\]

where \(Ar = \frac{\rho g x_{sv} \left(\frac{\rho}{\rho_p} - 1\right)}{\mu} \), Archimedes number

\[
Re_{mf} = \frac{U_{mf} x_{sv}}{\mu}
\]

= 0.4, usually

More practically,

- \(Wen\ and\ Yu\ (1966)\ for\ x_{sv} > 100\ m\)

\[
Ar = 1056 Re_{mf} + 159 Re_{mf}^2
\]

- \(Baeyens\ and\ Geldart\ (1974)\ for\ x < 100\ m\)

\[
U_{mf} = \frac{(\rho - \rho_p) \mu}{g} x_{sv}
\]

\[
= 1.13 \rho_p
\]

* Densities of particles
  - Absolute density: materials property
  - Particle density: Figure 5.2
  - Bed density

* Sieve diameter, \(x_p\), \(x_v = 1.13x_p\)

\[
\text{mean } x_p = \frac{1}{\sum m_i x_i}
\]

(2) Bubbling and Non-Bubbling Fluidization (5.3)

Types of Fluidization
Various types of fluidized beds

- **Bubbling fluidized bed**: Figure 5.3 for Group B particles

- **Liquid fluidization**: Figure 5.4

**Worked Example 5.1**

**Classification of Powders (5.4)**

Geldart(1974) Figure 5.6

*Table 5.1*
Group A : Nonbubbling for \( U_{mf} < U < U_{mb} \)

Group B : Bubbling for \( U > U_{mf} \)

No maximum in bubble size

Group D : Spoutable

Group C : Subject to channeling in large diameter-bed

(4) Applications of Fluidized Beds(5.8)

Advantages

- Liquid-like behavior, easy to control and automate
- Rapid mixing, uniform temperature and concentration
- Resists rapid temperature changes, hence responds slowly to changes in operating conditions and avoids temperature runaway with exothermic reactions
- Circulate solids between fluidized beds for heat exchange
- Applicable for large or small scale operations
- Heat and mass transfer rates are high, requiring smaller surfaces

Disadvantages

- Bubbling beds are difficult to predict and are less efficient
- Rapid mixing of solids causes nonuniform residence times for continuous flow reactors
- Particle comminution(breakup) is common
- Pipe and vessel walls erode to collisions by particles

1) Physical Processes

Drying / Mixing / Granulation / Coating / Heat exchanger/ Adsorption

Figure 5.17

2) Chemical Processes

Table 5.2

Figure 5.18 Fluidized catalytic cracker
4.3 Pneumatic Transport (Chapter 6)

(1) Pneumatic Transport

- Use of a gas to transport a particulate solid through pipeline

- Three major variables for pneumatic conveying
  - solid mass flow rate
  - gas mass flow rate
  - pressure gradient (pressure drop per unit length)

1) Dilute-Phase and Dense-Phase Transport

<table>
<thead>
<tr>
<th>Dilute-Phase</th>
<th>Dense-Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>High gas velocity (&gt; 20 m/s)</td>
<td>Low-gas velocity (1-5 m/s)</td>
</tr>
<tr>
<td>Low solids concentration</td>
<td>High solids concentration</td>
</tr>
<tr>
<td>(&lt; 1 % by volume)</td>
<td>(&gt; 30 % by volume)</td>
</tr>
<tr>
<td>Low pressure drop (&lt;5 mbar/m)</td>
<td>High pressure drop (&gt; 20 mbar/m)</td>
</tr>
<tr>
<td>Short-route, continuous transport(&lt; 10 ton/h)</td>
<td>Batch or semibatch transport</td>
</tr>
<tr>
<td>Capable under negative pressure</td>
<td></td>
</tr>
<tr>
<td>Particles behave as individuals</td>
<td></td>
</tr>
<tr>
<td>Fully suspended in gas</td>
<td>Not-fully suspended in gas</td>
</tr>
<tr>
<td>Fluid-particle : dominant</td>
<td>Much interaction between particles</td>
</tr>
<tr>
<td></td>
<td>and between particle and wall</td>
</tr>
</tbody>
</table>
2) **The Choking Velocity in Vertical Transport**

*Figure 6.1 - $\frac{\Delta p}{\Delta L}$ vs. $U$ (gas superficial velocity) at various solids flow flux $G$*  

**Static head of solids → friction resistance**

**Choking velocity, $U_{CH}$**

The lowest velocity at which the dilute-phase transport can operate at $G$ given

Punwani et al. (1976)

$$\frac{U_{CH}}{U_T} = G \rho (1 - \varepsilon_{CH})$$

$$0.77 \varepsilon = \frac{2250 D (\frac{4.7}{U_{CH}_T} - 1)}{[\frac{U_{CH}}{U_T}]^2}$$

3) **Saltation Velocity in Horizontal Transport**

*Figure 6.2 - $p/ L$ vs. $U$ (gas superficial velocity) at various solids flow flux $G$*  

**Saltation velocity, $U_{SALT}$**

The gas velocity at which the solids to begin to settle out  
Boundary between dilute phase flow and dense phase flow

Rizk (1973)

$$\frac{M_p}{U_{SALT} A} = \left\{ \frac{1}{10} \left( \frac{1}{1440x + 36} \right) \right\} \left\{ \frac{U_{SALT}}{\sqrt{\rho D}} \right\}^{(1100x + 2.5)} \text{ in SI}$$

solid loading  
Froude number  
at saltation

where $M_p$ : particle mass flow rate  
$D$ : pipe diameter
4) Fundamentals

Gas and particle velocity

**Superficial velocity**

\[ U_{fs} = \frac{Q_f}{A} \quad \text{and} \quad U_{fp} = \frac{Q_p}{A} \]

**Actual velocity**

\[ U_f = \frac{Q_f}{A} = U_{fs} \quad \text{and} \quad U_p = \frac{Q_p}{A(1-\varepsilon)} = \frac{U_{ps}}{1-\varepsilon} \]

* Slip velocity \( U_{slip} \)

\[ U_{rel} = U_f - U_p = U_{slip} \]

**Continuity**

Gas mass flow rate

\[ M_f = A U_f \]

Particle mass flow rate

\[ M_p = A U_p (1-\varepsilon) \]

**Solid loading**

\[ \frac{M_p}{M_f} = \frac{U_p (1-\varepsilon)}{U_f} \]

**Pressure drop**

From Newton's 2nd law of motion \ Figure 6.3

Rate of momentum for flowing gas-solid mixture

\[ \downarrow \quad p_1 - p_2 = \frac{1}{2} \rho_f U_f^2 + \frac{1}{2} \rho_s (1-\varepsilon) U_p^2 + F_{fwL} + F_{pwl} + \rho_f g \sin \theta + \rho_s (1-\varepsilon) g \sin \theta \]

5) Design for Dilute Phase Transport

Gas velocity
\[ U_f \sim 1.5 U_{SALT} \quad \text{since} \quad U_{SALT} > U_{CH} \]

for systems comprising both vertical and horizontal lines

\[ U_f \sim 1.5 U_{CH} \]

for vertical line only

Table. Approximate air velocity for powder transport

<table>
<thead>
<tr>
<th>Powder</th>
<th>( U, \text{ m/s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat, rice, plastic pellets</td>
<td>16 - 24</td>
</tr>
<tr>
<td>Grains, limestone powder</td>
<td>16 - 23</td>
</tr>
<tr>
<td>Soda ash, sugar</td>
<td>15 - 20</td>
</tr>
<tr>
<td>PVC powder</td>
<td>20 - 26</td>
</tr>
<tr>
<td>Carbon powder</td>
<td>18 - 24</td>
</tr>
<tr>
<td>Cement</td>
<td>18 - 28</td>
</tr>
<tr>
<td>Alumina powder</td>
<td>24 - 32</td>
</tr>
<tr>
<td>Sand</td>
<td>23 - 30</td>
</tr>
</tbody>
</table>

Pipeline pressure drop

\[ F_{pD}L = 0.057GL\sqrt{\frac{\rho}{D}} \quad \text{for vertical transport} \]

\[ F_{pD}L = \frac{2f_p(1 - \varepsilon)}{D} \frac{U^2_pL}{D} = \frac{2f_pG U^2_pL}{D} \quad \text{for horizontal transport} \]

where \( U_p = U_f(1 - 0.0638 x^{0.3} \rho^{0.5}) \) and

\[ f_p = \frac{3}{8} \frac{L}{D_p} CD \frac{D}{D_p} \left( \frac{U_f - U_p}{U_p} \right) \]

\( C_D \): drag coefficient (fn of \( Re_p \))

Bend

\( \sim 7.5 \text{ m of vertical section pressure drop} \)

* Downflow through vertical-to-horizontal bend:
  - greater tendency for saltation
  - avoided if possible.
* Blinded tee bend: Figure 6.4 with respect to radius elbow
  - prolonging service life due to cushioning effect
  - with the same pressure drop and solid attrition rate

Worked Example 6.1

Equipment

Figure 6.5 Positive pressure system
Figure 6.6 Negative Pressure system

* Centrifugal blowers(fan) vs. Positive displacement blower
  low pressure
  small amount of dust allowed
  high pressure
  no dust is allowed

Some problems in pneumatic transport

<table>
<thead>
<tr>
<th>Possible</th>
<th>Avoided by</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocking at high concentration</td>
<td>feeding at dispersed state sufficient acceleration length and adequate bend curvature</td>
</tr>
<tr>
<td>region(around solid feeder and</td>
<td></td>
</tr>
<tr>
<td>bend)</td>
<td></td>
</tr>
<tr>
<td>Adhesion with moisty, low-melting</td>
<td>adequate range of gas velocity</td>
</tr>
<tr>
<td>or electrically charged powder</td>
<td></td>
</tr>
<tr>
<td>Attraction at bend</td>
<td>- low gas velocity</td>
</tr>
<tr>
<td></td>
<td>- higher solid load</td>
</tr>
<tr>
<td></td>
<td>- changing collision angle and bend material.</td>
</tr>
</tbody>
</table>

6) Dense Phase Transport

Flow Patterns
- Horizontal - Figure 6.7
  Saltating flow - unstable, bad flow pattern
  Discontinuous dense phase flow*
**Dune Flow / Discrete Plug Flow / Plug Flow**

Continuous Dense Phase Flow - requires high pressure adequate for short-pipe transport

**Equipment**

Blow tanks: with fluidizing element (Figure 6.13)

without fluidizing element (Figure 6.14)

Plug formation: air knife (Figure 6.10)

air valve (Figure 6.11)

diaphragm (Figure 6.12)

Plug break-up: bypass (Figure 6.8)

pressure actuated valves (Figure 6.9)

**Design and Operation**

- Use of test facilities + past experience
  
  for pipe size, air flow rate and type of dense phase system

- Group A, D better than Group B, C for dense phase conveying

- Higher permeability: more suitable for plug flow type conveying

- Higher air retention: more suitable for dune mode flow

**4.3 Flow of Liquid-Solid Suspension (Slurries)(Supplement)**
**Characteristics of hydraulic transport**

**Transition velocity**

*Durand (1953)*

\[ U_t = 11.9 \left( \frac{U}{D} \right)^{1/2} \left( \frac{x}{D} \right)^{1/4} \]

where \( D \): pipe diameter

**Critical(saltation) velocity**

*Durand (1953)*

\[ U_c = F_L \left[ 2gD \left( \frac{\rho}{\rho_f} - 1 \right) \right]^{1/2} \]

where \( F_L \): function of \( x \) and \( \varepsilon \)

*Hanks (1980)*

\[ U_c = 3.12 \left( 1 - \varepsilon \right)^{0.386} \left( \frac{\rho}{\rho_f} \right)^{1/6} \left[ 2gD \left( \frac{\rho}{\rho_f} - 1 \right) \right]^{1/2} \]