CHBE320 LECTURE VIII
DYNAMIC BEHAVIORS OF CLOSED-LOOP CONTROL SYSTEMS

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Road Map of the Lecture VIII

- Dynamic Behavior of Closed-loop Control System
  - Closed-loop: controller is connected and working
  - Closed-loop transfer function
    - Response of output for set point change
    - Response of output for load/disturbance change
  - Effects of each block on closed-loop system
    - Effect of controller tuning parameters

Block Diagram Representation

- Standard block diagram of a feedback control system

  - Process TF: MV (M) effect on CV (X₂, part of Y)
  - Load TF: DV (L) effect on CV (X₁, part of Y)
  - Sensor TF: CV (Y) is transferred to measurement (B)
  - Actuator TF: Controller output (P) is transferred to MV (M)
  - Controller TF: Controller output (P) is calculated based on error (E)
  - Calibration TF: Gain of sensor TF, used to match the actual var.

- Individual TF of the standard block diagram
  - TF of each block between input and output of that block
  - Each gain will have different unit.
    - [Example] Sensor TF
      - Input range: 0-50 l/min
      - Output range: 4-20 mA
      - Dynamics: usually 1st order with small time constant

  - Block diagram shows the flow of signal and the connections
  - Schematic diagram shows the physical components connection
P&ID

- **Piping and Instrumentation diagram**
  - A P&ID is a blueprint, or map, of a process.
  - Technicians use P&IDs the same way an architect uses blueprints.
  - A P&ID shows each of the instruments in a process, their functions, their relationship to other components in the system.
  - Most diagrams use a standard format, such as the one developed by ISA (Instrumental Society of America) or SAMA (Scientific Apparatus Makers Association).
### General Instrument or function symbols

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1. Symbol size may vary according to the user’s needs and the type of document.
2. Abbreviations of the user’s choice may be used when necessary to specify location.
3. Inaccessible (behind the panel) devices may be depicted using the same symbol but with a dashed horizontal bar.

Source: Control Engineering with data from ISA S5.1 standard

- **Instrument description examples**
  - **FIC-101**: Flow Indicator and Controller, 0 to 50 m³/hr, (normal reading 30 T/hr). This instrument controls the flow of cold feedstock entering the tube side of the heat exchanger by positioning a valve on the cold feedstock flow path.
  - **FR-103**: Flow Recorder, 0 to 10 Ton/hr, (2.14 T/Hr). This instrument records the steam flow rate.
  - **HS-101**: Hand Switch, ON/OFF (ON). This switch turns on/off cold feedstock pump P-101. When the switch is in the ON condition, the pump is running. When the switch is in the OFF condition, the pump is not running.
  - **HV-102**: Hand Valve, OPEN/CLOSED, (OPEN). This switch opens/closes the steam block valve through which steam is routed from the header to the shell side of the heat exchanger. When the switch is in the OPEN condition the block valve is open. When the switch is in the CLOSED condition, the block valve is closed.
  - **PAL-103**: Pressure Alarm Low, (Normal). This alarm fires should the steam header pressure be less than 6 kg/cm².
- PI-100: Pressure Indicator, 0 to 15 kg/cm², (3.18 Kg/cm²). This instrument displays the steam pressure at the shell side of the heat exchanger.
- PI-103: Pressure Indicator, 0 to 15 kg/cm², (10.55 Kg/cm²). This instrument displays the steam header pressure.
- TAH/L-102: Temperature Alarm High/Low, (Normal). This alarm fires should the temperature of the feedstock at the exchanger outlet exceed 85°C or be less than 71°C.
- TI-103: Temperature Indicator, 0 to 200°C, (186°C). This instrument displays the temperature of the steam entering the shell side of the heat exchanger.
- TIRC-102 : Temperature Indicator, Recorder, and Controller, 0 to 200°C, (80°C). This instrument controls the temperature of the feedstock at the exchanger outlet by positioning the valve that regulates the steam flow to the exchanger.
- TR-101: Temperature Recorder, 0 to 200°C, (38°C). This instrument displays the temperature of the feedstock entering the exchanger.

CLOSED LOOP TRANSFER FUNCTION

- Block diagram algebra

\[
\begin{align*}
U(s) & \to X_1(s) \to \cdots \to X_n(s) \to Y(s) \\
\frac{Y(s)}{U(s)} & = X_1(s) - X_2(s)X_1(s) \\
Y(s) & = X_1(s)U_1(s) + X_2(s)U_2(s)
\end{align*}
\]

- Transfer functions of closed-loop system

\[
\begin{align*}
X_1(s) & = G_1(s)X_2(s)E(s) \\
E(s) & = K_1(s)R(s) - G_2(s)Y(s) \\
Y(s) & = G_1(s)E(s) + G_2(s)G_3(s)G_4(s)E(s) \\
Y(s) & = G_1(s)E(s) + G_2(s)G_3(s)G_4(s)E(s) - G_2(s)Y(s) \\
Y(s) & = (1 + G_2(s)G_3(s)G_4(s))Y(s) - G_2(s)Y(s) + K_1(s)G_2(s)G_3(s)G_4(s)E(s)
\end{align*}
\]

- For set-point change (L=0)

\[
\frac{Y(s)}{R(s)} = \frac{K_2G_1(s)G_4(s)G_5(s)}{1 + G_2(s)G_3(s)G_4(s)G_5(s)}
\]

- For load change (R=0)

\[
\frac{Y(s)}{L(s)} = \frac{G_2(s)}{1 + G_2(s)G_3(s)G_4(s)G_5(s)}
\]

- Open-loop transfer function (G_{OL})

\[
G_{OL}(s) = G_2(s)G_3(s)G_4(s)G_5(s)
\]

- Feedforward path: Path with no connection backward
- Feedback path: Path with circular connection loop
- G_{OL}: feedback loop is broken before the comparator

- Simultaneous change of set point and load

\[
\frac{Y(s)}{R(s) + L(s)} = \frac{K_2G_1(s)G_4(s)G_5(s)}{1 + G_2(s)G_3(s)G_4(s)G_5(s)}(R(s) + L(s))
\]
MASON’S RULE

- General expression for feedback control systems

\[ \frac{Y}{X} = \frac{n_f}{1 + n_e} \]

- Assume feedback loop has negative feedback.
- If it has positive feedback, \( n_f \) should be \( -n_e \).
- In the previous example, for set-point change

\[ \frac{Y}{X} = \frac{n_f}{1 + n_e} \]

- For load change, \( \frac{Y}{X} = G_{dl}(s) \)

Example 1

- Inner loop:
  \( \frac{Y}{R} = G_{c2}G_{c1} \)
  \( \frac{Y}{L} = G_{m2}G_{c1} \)

Example 2

- Inner loop:
  \( \frac{Y}{R} = G_{c2}G_{c1} \)
  \( \frac{Y}{L} = G_{m2}G_{c1} \)

PID CONTROLLER REVISITED

- P control
  \( p(t) = \beta + Ke(t) \)

- PI control
  \( p(t) = \beta + K_c \left( e(t) + \frac{1}{T_i} \int e(t)dt \right) \)

- PID control
  \( p(t) = \beta + K_c \left( e(t) + \frac{1}{T_i} \int e(t)dt + \frac{T_d}{T} \frac{de}{dt} \right) \)

- Ideal PID controller: Physically unrealizable
  - Modified form has to be used.
**Nonideal PID controller**

- **Interacting type**
  \[ G_e(s) = K_e \left( \frac{t_1 s + 1}{t_1 s + \beta t_2 s + 1} \right) \]

- **Comparison with ideal PID except filter**
  \[ G_e(s) = K_e \left( \frac{t_1 s + 1}{t_1 s + \beta t_2 s + 1} \right) \]
  \[ K_e = \frac{K_e^*(t_1 s + \beta t_2 s + 1)}{t_1 s + \beta t_2 s + 1} \]

  - These types are physically realizable and the modification provides the prefiltering of the error signal.
  - Generally, \( t_1 \geq t_0 \) and typically \( t_1 \approx 4t_0 \).
  - In this form, \( t_1 \geq t_0 \) is satisfied automatically since algebraic mean is not less than logarithm mean.

**Other variations of PID controller**

- **Gain scheduling : modifying proportional gain**
  \[ K_{eG} = K_e K_{eG} \]
  where
  1. \( K_{eG} = K_{eG} \) for \((\text{lower gap}) \leq e(t) \leq (\text{upper gap})\)
  2. \( K_{eG} = 1 + C_{eG} |e(t)| \)
  3. \( K_{eG} \) is decided based on some strategy

- **Nonlinear PID controller**
  - Replace \( e(t) \) with \( |e(t)| \).
  - Sign of error will be preserved but small error gets smaller and larger error gets larger.
  - It imposes less action for a small error.

**Block diagram of PID controller**

- **Nonideal interacting type PID**
  \[ E(s) \]
  \[ P(s) = K_c (r(t) + 1) \]

  - **Removal of derivative kick (PI-D controller)**
  \[ P(s) = K_c (r(t) + 1) \]

  - **Removal of both P & D kicks (I-PD controller)**
  \[ P(s) = K_c \]

**DIGITAL PID CONTROLLER**

- **Discrete time system**
  - Measurements and actions are taken at every sampling interval.
  - An action will be held during the sampling interval.

- **Digital PID controller**
  - Using \( \int_{0}^{t} e(t) dt + I \sum_{n=0}^{\infty} e(t_n) \) (Rectangular rule)
  - \( \frac{de(t)}{dt} = e(t_n) - e(t_{n-1}) \) (Backward difference approx.)
  \[ P(t_n) = \hat{p} + K_c \left( e(t_n) + \frac{\Delta t}{t_1} e(t_{n-1}) + \frac{\Delta t}{t_2} e(t_{n-2}) \right) \]
  \[ \Delta e(t_n) = e(t_n) - e(t_{n-1}) \]
  \[ \Delta e(t_n) = e(t_n) - e(t_{n-1}) + \frac{\Delta t}{t_1} e(t_n) + \frac{\Delta t}{t_2} e(t_{n-1}) - 2e(t_{n-1}) + e(t_{n-2}) \] (Velocity form)
Most modern PID controllers are manufactured in digital form with short sampling time.

If the sampling time is small, there is not much difference between continuous and digital forms.

Velocity form does not have reset windup problem because there is no summation (integration).

Other approximation such as trapezoidal rule and etc. can be used to enhance the accuracy. But the improvement is not substantial.

For discrete time system, z-transform is the counterpart of Laplace transform. (out of scope of this lecture)

\[
\int_{t}^{t+\Delta t} e(t)dt = \Delta t \sum_{i=0}^{\infty} \frac{e(t_i) + e(t_{i-1})}{2} \quad \text{(Trapezoidal rule)}
\]

\[
\frac{de(t)}{dt} = \frac{e(t) + 3e(t_{i-1}) - 3e(t_{i-2}) - e(t_{i-3})}{\Delta t} \quad \text{(Interpolation formula)}
\]

- **P control for set-point change (L=0)**
  \[G_c(s) = K_c \quad (K_c > 0)\]
  \[G_{cl}(s) = \frac{H(s) + \frac{K_c K_p}{(1 + K_c K_p)}(s + 1)}{1 + \frac{K_c K_p}{(1 + K_c K_p)}(s + 1)} \quad \text{(closed-loop TF)}\]

  - Closed-loop gain and time constant
    \[K_{cl} = \frac{K_c K_p}{(1 + K_c K_p)} \quad \tau_{cl} = \frac{1}{(1 + K_c K_p)}\]

  - Steady-state behavior of closed-loop system
    \[K_{cl} = \frac{K_c K_p}{(1 + K_c K_p)} < 1, \quad \lim_{s \to 0} G_{cl} = 1 \quad \text{(H(s) = R(s), no offset)}\]

- **P control for load change (R=0)**
  \[G_c(s) = K_c \quad (K_c > 0)\]
  \[G_{cl}(s) = \frac{H(s) + \frac{K_p}{(1 + K_c K_p)}(s + 1)}{1 + \frac{K_p}{(1 + K_c K_p)}(s + 1)} \quad \text{(closed-loop TF)}\]

  - Closed-loop gain and time constant
    \[K_{cl} = \frac{K_p}{(1 + K_c K_p)} \quad \tau_{cl} = \frac{1}{(1 + K_c K_p)}\]

  - Steady-state behavior of closed-loop system
    \[K_{cl} = \frac{K_p}{(1 + K_c K_p)} > 0, \quad \lim_{s \to 0} G_{cl} = 0 \quad \text{(disturbance is compensated)}\]
• PI control for load change (R=0)

\[ G_l(s) = \frac{K_c}{(s+1)} \]

- Closed-loop gain, time constant, damping coefficient

\[ K_{num} = \frac{\tau_l}{K_c} \quad \tau_l = \frac{\tau_c}{K_c} \quad \zeta_l = \frac{1 + K_c}{\sqrt{K_c K_p}} \frac{\tau_c}{\tau_l} \]

- Steady-state behavior of closed-loop system

\[ \lim_{s \to 0} G_l(s) = 0 \] (disturbance is compensated for all cases)

- As \( K_c \) increases, faster compensation of disturbance and less oscillatory response can be achieved.
- As \( \tau_c \) decreases, faster compensation of disturbance and less overshooting response can be achieved.
- However, usually the response gets more oscillation as \( K_c \) increases or \( \tau_c \) decreases. >> very unusual!!
- If there is small lag in sensor/actuator TF or time delay in process TF, the system becomes higher order and these anomalous results will not occur. These results is only possible for very simple process such as 1st order system.
- Usual effect of PID tuning parameters
  - As \( K_c \) increases, the response will be faster, more oscillatory.
  - As \( K_c \) decreases, the response will be faster, more oscillatory.
  - As \( K_c \) increases, the response will be faster, less oscillatory when there is no noise.

CLOSED-LOOP RESPONSE OF INTEGRATING SYSTEM

- Process

\[ \rho A \frac{dh}{dt} = \rho(\dot{q}_1 + \dot{q}_2) - \rho q_1 \]

\[ G_c(s) = \frac{H(s)}{Q_1(s)} = -\frac{1}{A} \]

\[ G_l(s) = \frac{H(s)}{Q_2(s)} = \frac{1}{A} \]

- Assume

Sensor and actuator dynamics are fast enough to be ignored and gains are lumped in other TF.

\[ G_c(s) = G_m(s) = 1 \]

\[ H(s) = \frac{G_c G_p}{1+G_c G_p} R(s) + \frac{G_l}{1+G_c G_p} L(s) \]

- As \( K_c \) increases, faster compensation of disturbance and less oscillatory response can be achieved.
- As \( \tau_c \) decreases, faster compensation of disturbance and less overshooting response can be achieved.
- However, usually the response gets more oscillation as \( K_c \) increases or \( \tau_c \) decreases. >> very unusual!!
- If there is small lag in sensor/actuator TF or time delay in process TF, the system becomes higher order and these anomalous results will not occur. These results is only possible for very simple process such as 1st order system.
- Usual effect of PID tuning parameters
  - As \( K_c \) increases, the response will be faster, more oscillatory.
  - As \( K_c \) decreases, the response will be faster, more oscillatory.
  - As \( K_c \) increases, the response will be faster, less oscillatory when there is no noise.

- P control for set-point change (L=0)

\[ G_c(s) = K_c \quad (K_c<0) \]

\[ G_l(s) = \frac{H(s)}{R(s)} = \frac{K_c/(-A)}{1+K_c/(-A)} = \frac{1}{(1+K_c/(-A))s + 1} \] (close-loop TF)

- Closed-loop gain and time constant

\[ K_{cl} = 1 \quad \tau_{cl} = -A/K_c \]

- Steady-state behavior of closed-loop system

\[ K_{cl} = 1 \quad (H(s) = R(s), no offset even with p control) \]

- It is very unique that the integrating system will not have offset even with P control for the set point change.
- Even though there are other dynamics in sensor or actuator, the offset will not be shown with P control for integrating systems.
- Higher controller gain results faster closed-loop response: shorter time constant.
• P control for load change (R=0)

\[ G_c(s) = K_c \quad (K_c < 0) \]

\[ G_{cl}(s) = H(s) \cdot \frac{1/(\tau c)}{1 + K_c(\tau c)} = \frac{-1/K_c}{1 - \frac{A}{K_c} s} \quad \text{(closed-loop TF)} \]

- Closed-loop gain and time constant

\[ K_c = (-1)/K_c, \quad \tau_c = -A/K_c \]

- Steady-state behavior of closed-loop system

\[ K_c = \frac{1}{(-K_c)} > 0, \quad \lim_{s \to -\infty} G_{cl} = 0 \quad \text{(disturbance is compensated)} \]

- Higher controller gain results faster closed-loop response: shorter time constant

\[ G_{cl}(s) = H(s) \cdot \frac{1/(\tau c)}{1 + K_c(\tau c)} = \frac{-1/K_c}{1 - \frac{A}{K_c} s} \]

\[ \lim_{s \to -\infty} G_{cl} = 0 \quad \text{(disturbance is compensated)} \]

• PI control for set-point change (L=0)

\[ G_c(s) = K_i(\tau i s + 1)/\tau i s \quad (K_i < 0) \]

\[ G_{cl}(s) = \frac{K_i(\tau i s + 1)/\tau i s}{1 + K_i(\tau i s + 1)/(-A/\tau i s)} = \frac{(\tau i s + 1)}{1 + \frac{A}{\tau i s} + \frac{1}{\tau i s}} \]

- Closed-loop gain, time constant, damping coefficient

\[ K_i = 1, \quad \tau_i = \sqrt{\frac{\tau i}{A}}, \quad \zeta = \frac{1}{2} \sqrt{\frac{\tau i K_i}{A}} \]

- Steady-state behavior of closed-loop system

\[ K_i = \lim_{s \to 0} \frac{G_{cl}(s)}{s} = 1 \quad \text{(H(s) = R(s), no offset)} \]

- As (-Kc) increases, closed-loop time constant gets smaller (faster response) and less oscillatory response can be achieved.
- As \( \tau_i \) decreases, closed-loop time constant gets smaller (faster response) and more oscillatory response can be achieved.
- Partly anomalous results due to integrating nature
- For integrating system, the effect of tuning parameters can be different. Thus, rules of thumb cannot be applied blindly.