Flow between eccentric cylinders: a shear-extensional controllable flow

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In this work the non-Newtonian fluid between eccentric cylinders is simulated with finite element method. The flow in the annular gap between the eccentric rotating cylinders was found to be a shear-extensional controllable flow. The influence of rotating speed, eccentricity as well as the radius ratio on the extensional flow in the vicinity of the minimum gap between the inner and outer cylinder was quantitatively investigated. It was found that both the strengths of shear flow and extensional flow could be adjusted by changing the rotating speed. In respect to extensional flow, it was also observed that the eccentricity and radius ratio exert significant influences on the ratio of extensional flow. And it should be noted that the ratio of extensional flow in the mix flow could be increased when increasing the eccentricity and the ratio of shear flow in the mix flow could be increased when increasing the radius ratio.

Keywords: extensional flow, shear flow, numerical simulation, flow between eccentric cylinders, velocity distribution

1. Introduction

The melt flow in polymer processing is a mixture of shear flow and extensional flow. For instance, the flow in the injection moulds where the cross-section is varying and the flow between twin-screw elements in extrusion (Yanovsky, 2009). The strength and ratio of both flows exert significant influence on the micro structure (i.e., the orientation of macromolecule and the dispersion state) which affects the macro properties of the end products (i.e., size, shape, and mechanical properties). Therefore, regulating the strength and ratio of both flows makes it possible to tailor the final properties of the product. Currently, the ratio of extensional flow in most polymer processing equipment is much less than that of shear flow. However, considerable evidence has been accumulated to demonstrate that extensional flow in polymer processing has many advantages (Qu et al., 2013). And several methods have been found to generate extensional flow field. Bauwendaal et al. (1999) introduced a new generation of dispersive mixing elements used in internal mixers, single or twin screw extruders. It suggested that the dispersive mixing elements contributed to the increase of ratio of extensional flow in the mixture flow. Wei et al. (2007) measured extensional viscosities with a capillary rheometer in which the capillary cylindrical die was replaced by a hyperbolic converging die. A purely extensional flow field was established by the hyperbolic converging die at a constant extensional strain rate throughout the die. Bouquey et al. (Bouquey et al., 2011; Ibarra-Gómez et al., 2015; Rondin et al., 2014) developed a novel laboratory-scale mixing device in which the flow was characterized by a high contribution from extensional flow. Recently, in order to create strong extensional flow during polymer processing, Qu et al. (Jia et al., 2014; Qu et al., 2012a; Qu et al., 2012b) in our group invented a novel polymer processing equipment without a screw known as vane extruder, which consisted of a number of vane plasticizing and conveying units. And the vane extruder can generate higher stress and dynamic extensional flow in the whole process by continuous dynamical converging channels. Although the above-mentioned devices or equipment can create considerable extensional flow, they can not regulate the strength and ratio of shear flow and extensional flow.

The flow between eccentric rotating cylinders is a mixture of shear and extensional flows (Boonen et al., 2010). The flow between eccentric rotating cylinders is of considerable technological importance and as a result, has received much attention and been studied extensively in fluid mechanics (Dris and Shaqfeh, 1998a; 1998b). Ballal and Rivlin (1962) first established the theoretical model of a Newtonian flow in the annular region between eccentric cylinders. Li (2014) simulated the flow between eccentric cylinders with different viscoelastic constitutive equations. Results indicated that viscoelasticity did enhance the lubricant pressure field and produce a beneficial effect on lubrication performance. Larson et al. (1990) found that Couette flow would translate into T–C flow (Taylor-Couette flow) and instability occurred at low rotating speed when the flow was elastic. Dris and Shaqfeh (1996, 1998a,
1998b) further investigated the viscoelastic flow between the eccentric cylinders both numerically and experimentally. It was pointed out that the combination of large elastic normal stresses and streamline curvature was an essential factor to drive elastic instabilities. Grecov and Clermont (2002, 2005, 2008) applied the stream-tube method (STM) to the simulation of unsteady flows of non-Newtonian incompressible fluids between concentric and eccentric cylinders. Rajagopalan et al. (1992) attempted to use the birefringence method which was based on stress optical principle to investigate the state of stress between eccentric cylinders. It was found that the measurement principle was applicable to pure shear flow, for shear-extensional flow the principle fails. Feigl et al. (2003) investigated the breakup behavior of drops in the annular gap flow between two eccentric cylinders in which the inner cylinder rotated at a constant speed. And they introduced a coefficient to define the ratio of extensional deformation rate in the complex flow field, but this coefficient has not been applied to the actual flow field yet. Boonen et al. (2009, 2010) newly designed eccentric cylinder device which was equipped with a microscope and camera system to study the deformation and orientation of single Newtonian droplets dispersed in a Newtonian matrix undergoing controlled mixed flow conditions. It was observed that the rotating styles (i.e., outer cylinder rotating, counter-rotating, or co-rotating) had a profound effect on the type of mixed flow obtained. The flow conditions were mostly shear dominated only if the outer cylinder was rotating at a constant speed, while the extensional flow increased drastically when both cylinders were co-rotating at different speeds. It was proved that the rotating style of the inner and outer cylinders was also an important method to control the flow field.

This is the preparation work of the development of a novel shear-extensional rheometer in which the shear flow and extensional flow are controllable based on the flows between eccentric cylinders. As is known, the development of a novel rheometer is a demanding task including a lot of mathematic modeling, precision machinery, and automation. Therefore, it is necessary to investigate the flow between eccentric cylinders in a numerical way which could help to facilitate the development of our target equipment. In the present study, the strength and ratio of extensional flow between eccentric cylinders have been investigated when changing the rotating speed, eccentricity and radius ratio. The Bird-Carreau model is applied to simulate the non-Newtonian flow between eccentric cylinders, which includes the distribution of velocity and strain rate in the flow field. And the paper is organized as follows. In sections 2 and 3, we present the Bird-Carreau model and the eccentric cylinder configuration. In section 4, we present a discussion of numerical simulations with various rotating speed, eccentricity, and radius ratio. Finally, some concluding remarks are provided in section 5.

2. The Mathematical Model

Here we first state the mathematical model for non-Newtonian fluid and begin with the conservation laws. For the situations we will address, the polymeric fluid in isothermal annular gap flow between the eccentric cylinders can be assumed to be incompressible. The governing equations of continuity and momentum conservations are given by

\[ \nabla \cdot \mathbf{u} = 0 , \]

\[ \nabla \cdot \sigma = 0 . \]

Here \( \mathbf{u} \) denotes the velocity field while \( \sigma \) denotes the Cauchy stress tensor. In addition, we simulate the non-Newtonian flow between eccentric cylinders by the generalized Newtonian model. The Bird-Carreau model (Domurath et al., 2015; Hosseinalipour et al., 2013) was selected in order to describe the rheological behavior of the non-Newtonian flow between eccentric cylinders. For generalized Newtonian fluids, the stress tensor is given as

\[ \sigma = \eta (\dot{\gamma}) \dot{\gamma} \]

where \( \dot{\gamma} \) is the traceless rate-of-strain tensor and can be expressed as

\[ \dot{\gamma} = \nabla \mathbf{u} + (\nabla \mathbf{u})^T \]

where \( \mathbf{u} \) is the velocity vector, \( \nabla \mathbf{u} \) and \( (\nabla \mathbf{u})^T \) are the velocity gradient and transpose of the velocity gradient, respectively. For a Bird–Carreau fluid the apparent viscosity is given by

\[ \eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty)(1 + \lambda^2 \dot{\gamma}^2)^{n-1} \]

where \( \eta_\infty \) is the infinite-shear viscosity, \( \eta_0 \) is the zero-shear viscosity, \( \lambda \) is the characteristic time, and \( n \) is the power-law index.

2.1. Rheological parameters

The polymer material investigated here is LDPE from SINOPEC. In this paper, the model parameters of \( \eta_0 = 4.679 \times 10^5 \) Pa·s, \( \lambda = 1.3363 \) s, \( n = 0.3804 \), and \( \eta_\infty = 0 \). Those rheological parameters are obtained by fitting the model parameters to the experimental data.

3. Numerical Simulation

3.1. Flow geometry

In this study, fluid flows in the annular gap between the eccentric rotating cylinders whose axes are parallel, but offset by a distance \( e \) (mm), and the \( e \) (mm) is defined as eccentricity. The outer cylinder is stationary with a radius of various rotating speed, eccentricity, and radius ratio. Finally, in sections 2 and 3, we present the Bird-Carreau model and the eccentric cylinder configuration. In section 4, we present a discussion of numerical simulations with various rotating speed, eccentricity, and radius ratio. Finally, the outer cylinder is stationary with a radius of some concluding remarks are provided in section 5.

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whereas the inner cylinder with a radius $R_i$ and rotates at a constant angular velocity $\omega$ which will change in the range of 0-200 rpm. The origin of the cylindrical polar coordinate system is taken at the center of the inner cylinder. The azimuthal coordinate, $\theta$ is defined so that the minimum gap is at $\theta = 180^\circ$. In Fig. 1 we display a typical view of the geometry and parameters.

3.2. Convergence angle

Different from the general planar contraction flow, the convergence angle is not constant in the annular gap between the eccentric rotating cylinders, and it can be adjusted by changing eccentricity. It is notable that the value of convergence angle is zero when $\theta = 0^\circ$ (or $180^\circ$). In the theory of convergent flow, the magnitudes of the convergence angle affect the extensional flow significantly in the annular gap between the eccentric rotating cylinders. For example, in Cogswell model (Cogswell, 1972a; 1972b), the average extensional stress and the average extensional strain rate are the function of the tangent value of the convergence semi-angle. Therefore, it is tempting to conclude that the magnitude of the convergence angle has a great influence on the extensional rheological behavior. Fig. 2 shows the domain between the two eccentric cylinders. The parameter $e$ denotes the distance between the axes of the cylinders. According to the triangle relationship between the corners we can obtain that the convergence angle $\alpha$ is equal to $\beta$. In the triangle which is consist of line a, line b and line $O_iO_2$, we can utilize the sine theorem, according to the relation

$$
\frac{\sin \beta}{e} = \frac{\sin \theta}{R_o}.
$$

So we obtain,

$$
\beta = \arcsin \left( \frac{e}{R_o} \sin \theta \right), \quad (0 < e < R_o - R_i).
$$

Fig. 3 shows the distribution of convergence angle at the different eccentricities, for the radius of $R_o = 25$ mm, $R_i = 20$ mm, and eccentricity of $e = 1$ mm, 2 mm, 3 mm, and 4 mm, respectively. When $\beta > 0$, the annular gap between the eccentric cylinders grows narrower with the $\theta$ and the melt experiences prominent extensional flow. On the contrary, when $\beta < 0$, the annular gap between the eccentric cylinders grows wider and the melt experiences prominent contraction flow. Maximum convergence angle (divergence angle) appeared when $\theta = 90^\circ$ (or $270^\circ$). As seen in Fig. 3, when the eccentricity rises, the convergence angle increases accordingly. Consequently, we can adjust the size of the convergence angle (divergence angle) by changing the eccentricity continuously, and in mechanical design the continuous adjustment of the extensional-shear flow between the eccentric rotating cylinders can be achieved.

3.3. Computational technique

The simulation process is carried out using CFD soft-
ware ANSYS POLYFLOW with finite element method. The current Bird-Carreau model has been implemented into POLYFLOW. To simplify the problem, we assume that the polymer melt is incompressible and the inertial effects are neglected; temperature changes and gravity are also negligible. The subtask is assigned to be a generalized Newtonian isothermal flow problem in order to reduce calculation cost.

Before simulation, the computational mesh for the annular gap between the eccentric cylinders must be created. The density of the computational mesh has an effect on the accuracy of the simulation results. Generally speaking, high mesh density would lead to more accurate results but also with higher computation cost. Therefore, a reasonable computational mesh in order to ensure the reasonable accuracy and reasonable calculation time is needed. Fig. 4 is a close-up view of quadrilateral meshes applied in our simulation work. Detailed mesh information is presented in Table 1 (We can find the quadrilateral meshes have a tiny increment. It is just due to the different eccentricities when we construct the computational mesh by the same way because the numerical results change little, the tiny increment not be ignored). In fact, mesh refinement has been carried out for the area near the inner cylinder. Those meshes are chosen because the numerical results change little when the mesh refinement continues.

3.4. Reliability analysis
In order to validate the reliability of the simulations, we compared our simulation results with experiments from reference (Ballal and Rivlin, 1975; Dris and Shaqfeh, 1996). The numerical and experimental velocity profiles along the radial direction at the minimum gap are shown in Fig. 5, where \( V_\theta^* = V_\theta / (\omega R) \). It is easy to see the velocities which are measured by experiments fluctuates. This could be attributed to the prominent acceleration in the upstream of the minimum gap. However, within experimental error, the velocity profile predicted by Bird-Carreau model is in agreement with the profile measured by experiments. Therefore, comparison between experiment-

![Fig. 4.](image)

**Fig. 4.** (Color online) Schematic diagram of quadrilateral meshes between the eccentric cylinders \((R_o = 25 \text{ mm}, R_i = 20 \text{ mm}, e = 2 \text{ mm})\).

<table>
<thead>
<tr>
<th>Eccentricity ( e ) (mm)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
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<tr>
<td>Number of nodes</td>
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<td>12300</td>
<td>12464</td>
<td>12464</td>
<td>12464</td>
</tr>
</tbody>
</table>

![Table 1.](image)

**Table 1.** The information of the quadrilateral meshes at different eccentricities.

![Fig. 5.](image)

**Fig. 5.** (Color online) Velocity distribution along radius at the minimum gap for various eccentricities: (a) \( e^* = 0.2 \); (b) \( e^* = 0.6 \). \( (e^* \) is illustrated in Eq. (11)).
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tal and numerical methods shows that Bird-Carreau model is able to predict the distribution of velocity at the gap between eccentric cylinders.

4. Results and Discussion

In this paper, the important parameters are made dimensionless as follows:

Circumferential velocity:

\[ V'_\theta = V_\theta / (\omega R_i) . \]  

Radial velocity:

\[ \dot{y}' = \dot{y} / \omega , \]  

\[ \dot{e}' = \dot{e} / \omega . \]  

Eccentricity:

\[ \dot{e}' = e / b = e / (R_i - R_o) . \]  

Circumferential angle:

\[ \theta' = \theta / 2\pi . \]  

Radius ratio:

\[ \kappa = R_i / R_o . \]  

The distance between inner cylinder and any point in the flow:

\[ \delta^* = (r - R_i) / (R_o - R_i) . \]  

4.1. Effect of the rotating speed on melt flow

Velocity is a very important parameter in the flow field, which is closely related to the melt flow rate and the strain rate. In this study, we focus on the distribution of the velocity in the radial direction \((r)\) and circumferential direction \((\theta)\). Fig. 6 presents the simulations for the circumferential velocities and the extensional rates, as a function of the circumferential angle, \(\theta^*\), and the rotating speed, \(\omega\). And it is easy to see that the circumferential distribution of velocity is basically the same at different inner cylinder rotating speeds when the parameters of flow geometry are unchanged. At the maximum gap the velocity reaches minimum value and then increase slightly with \(\theta^*\), a sharp increase starts at \(\theta^* = 0.2\) and attains the max-

![Fig. 6.](image1)

![Fig. 7.](image2)
imum velocity at $\theta' = 0.5$ (the minimum gap). In this process, due to the convergence of the annular gap the melt experiences certain strength of extensional flow (shown in Fig. 6b). Afterwards, the velocity begins to decrease sharply and declines slowly in the vicinity of $\theta' = 0.8$, and the melt experiences a certain strength of contraction flow (see in Fig. 6b). The velocity curve and extensional rate curve almost remain unchanged when the inner cylinder rotating speed changed. Specific summaries of those important aspects are given below: a. the circumferential distribution of the velocity and the extensional rates will not change with the change of the rotating speed in eccentric cylinders; b. the circumferential velocity and the extensional rates mainly depends on the inner cylinder rotating speed, that is, $V_{\theta}' \sim \omega$ and $\dot{\varepsilon} \sim \omega$.

Velocity distribution along radius and the shear rate at the minimum gap at various rotating speeds are shown in Fig. 7. With the increase of $r^*$, the melt velocity decreases greatly, and reduces to zero on the outer cylinder surface. As seen in Fig. 7b, the minimum shear rate is obtained at the minimum gap on inner cylinder surface. Away from the inner cylinder surface, the shear rate increased near linearly with the increase of $r^*$ and the maximum shear rate is obtained on the outer cylinder surface. Because the extensional flow between the eccentric cylinders can make the radial velocity gradient smaller near the inner cylinder. The radial velocity curve and the shear rate curve are almost the same when the inner cylinder rotating speed changed. The main conclusions are listed below: a. the radial distribution of velocities and shear rates will not change with the change of the inner cylinder rotating speed in eccentric cylinders; b. the radial velocity and the shear rates mainly depend on the inner cylinder rotating speed, namely, $V_r' \sim \omega$ and $\dot{\gamma} \sim \omega$.

It can be concluded by the analysis of velocities that we can control the strength of the extensional flow and shear flow, and the flow velocity between the eccentric cylinders by adjusting the rotational speed of inner cylinder. But since the extensional rate and shear rate increase linearly as the inner cylinder rotating speed increases, changing the rotating speed of inner cylinder can effectively regulate the strength of the extensional-shear mixture flow while fails to regulate the proportion of the extensional flow and the shear flow.

4.2. Effect of the eccentricity on melt flow

Fig. 8 shows how the circumferential velocities and the extensional rates vary over a range of eccentricities. It is obvious that whilst the overall forms and trends of the profiles are similar, the absolute magnitudes for the velocity do not agree. With the change of the $\theta'$ (0-0.5), the velocity increases slightly and reaches the maximum value when $\theta' = 0.5$. In this process, the melt experiences a certain degree of extensional flow. And the velocity declines slightly to the minimum value when the $\theta'$ increases from 0.5 to 1. At the same time, the melt experiences a certain degree of contraction flow. When the eccentricity increases, the velocity shoots up before the melt flows through the minimum gap between the eccentric cylinders, and plunges after flowing through the minimum gap. On the other hand, the velocity experiences a modest decline with the eccentricity increase in the vicinity of the maximum gap. As seen in Fig. 8b, the extensional rates increase drastically with the increase of eccentricity when the melt flows through the upstream of the minimum gap. The maximum extensional rates at various eccentricities at the upstream of the minimum gap are shown in Fig. 9. It is easy to see that the maximum extensional rates soar which is similar to the curve of a quadratic equation.

Fig. 10 displays the velocity distribution along radius and the shear rate at the minimum gap at various eccentricities. As seen in Fig. 10a, it is easy to see that the velocities at both walls of inner and outer cylinders keep unchanged. And it could also be seen that, as $\delta^*$ increases, the melt velocity decreases. In addition, as the eccentricity rises, the velocity increases significantly at the annular.

**Fig. 8.** (Color online) Velocity distribution and the extensional rate distribution along circumferential at various eccentricities ($\delta^* = 0.5$, $\omega = 200$ rpm and $\kappa = 0.6$).
gap between the eccentric cylinders especially near the center of the two cylinders. When the two cylinders are concentric, the velocity curve in Fig. 10 is concave, whereas, it gradually turns into a convex curve when the eccentricity increases. And it is worth noting that the velocity is a linear function of the $\delta^*$ when $e^* = 0.6$. This transformation could be attributed to the rise of convergence angle when the eccentricity increases. And when the convergence angle rises, the acceleration effect tends to be stronger. Extensional flow is most prominent near the center of the flow which leads to the velocity increase sharply (see Fig. 10a). While the flow is pure shear flow near the inner and outer cylinder surfaces therefore, melt velocity in this area does not change much. The distribution of the shear rate is depicted in Fig. 10b. In agreement with the former velocity analysis, when $e^* < 0.6$, the shear rate along the radial direction gradually decreases and the minimum shear rate is obtained on the outer cylinder surface. When $e^* > 0.6$, the shear rate along the radial direction gradually increases and the maximum shear rate is obtained on the outer cylinder surface. And the shear rate is a constant approximately along the radial direction when $e^* = 0.6$.

According to the analysis above, the strength of the shear flow and extensional flow at the gap between the eccentric cylinders will change as the eccentricity changes. And the extensional rates will increase significantly with the increase of eccentricity. In order to quantify the ratio of extensional flow in the mix flow, We introduce a coefficient ($\phi$), a dimensionless group that indicates the ratio of two flows (Feigl et al., 2003). Here, the coefficient ($\phi$) expresses the relative amount of extension presented in the flow which is defined as:

$$
\phi = \frac{\dot{\varepsilon}}{\dot{\gamma}}
$$

where $\dot{\varepsilon}$ is the extensional rate in 2D plane, while $\dot{\gamma}$ stands for shear rate. In general, $-1 \leq \phi \leq 1$.

The ratio of extensional rate in the mix flow as a function of various eccentricities at the position of the maximum extensional rate is shown in Fig. 11. We can find that

$$
\phi = \frac{\dot{\varepsilon}}{\left| \dot{\gamma} \right| + \left| \dot{\gamma} \right|}
$$

Fig. 9. The extensional rates at various eccentricities at the upstream of the minimum gap.

Fig. 10. (Color online) Velocity distribution along radius and the shear rate at the minimum gap at various eccentricities ($\omega = 200$ rpm and $\kappa = 0.6$).

Fig. 11. The ratio of extensional rate in the mix flow at various eccentricities.
the extensional rate increases dramatically at first and then the slope of the curve decreases slightly. Actually, it can enhance the strengths of extensional rate by increasing eccentricity. This result is similar to that by increasing the rotating speed of the inner cylinder. From Figs. 6 and 8, it shows that the extensional rate is more sensitive to eccentricity. Unlike changing the rotating speed of the inner cylinder, we can control not only the strength of extensional flow and shear flow but also the ratio of the extensional-shear flow by changing the eccentricity.

4.3. Effect of the radius ratio on melt flow

The circumferential velocities and the extensional rates along with simulations from the BC models for the PE melt are shown in Figs. 12a and 12b, respectively. It is obvious that whilst the overall forms and trends of the profiles are similar and the absolute magnitudes for the velocities do not agree at various radius ratios. With the change of the $\theta^*$ (0-0.5), the velocities increase gradually and the maximum velocity is obtained when $\theta^* = 0.5$. In this process, the melt experiences a certain degree of extensional flow. And the velocity declines gradually in the range varying from $\theta^* = 0.5$ to $\theta^* = 1$ and the minimum velocity is obtained at $\theta^* = 1$. At the same time, the melt experiences a certain degree of contraction flow. The velocity increases as the radius ratio increases (that is, the radii of inner and outer cylinders are nearly equal). It is because the velocity distribution along radius has some changes with the change of radius ratio.

As seen in Fig. 12b, it is observed that the circumferential distribution of extensional rate almost remains unchanged at various radius ratios when the parameters of flow geometry are settled. And in the sector near the minimum gap, the melt experiences the maximum extensional flow in the upstream and experiences the maximum contraction flow in the downstream. It is worth mentioning that the extensional rate decreases almost linearly in the range varying from $\theta^* = 0.38$ to $\theta^* = 0.58$. The maximum extensional rates at various radius ratios are shown in Fig. 13. It should be noted that the maximum extensional rate increases with the increase of radius ratio. Because the velocity of the melt increases with the increase of radius ratio and the volume flow rate increases accordingly. It is similar to the planar contraction flow, the extensional rate between the eccentric cylinders can increase when the volume flow rate increases. And we can also find that the extensional rate increases dramatically at first and then increases slowly with the increase of radius ratio. And it is indicated that the extensional rate is less sensitive to radius ratio when the radii of inner and outer cylinders are nearly equal.

The profiles of the radial velocities and shear rates at various radius ratios at the minimum gap ($\theta^* = 0.5$) are depicted in Fig. 14. As seen in Fig. 10a, it is found that the maximum melt velocity is obtained on the inner cylinder surface and equal to the inner cylinder rotating speed. Away from the inner cylinder surface, the melt velocity decreases with the increase of $\delta^*$. In addition, the velocity at inner and outer cylinders wall keep fixed, and the velo-

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Fig. 12. (Color online) Circumferential distribution of the velocities and the extensional rates at various radius ratios ($\delta^* = 0.5$, $\omega = 200$ rpm, and $e^* = 0.6$).

Fig. 13. Plots of the maximum extensional rate at various radius ratios.
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Fig. 14. (Color online) Velocity distribution along radius and the shear rate at the minimum gap at various radius ratios ($\omega = 200$ rpm and $e^* = 0.6$).

Fig. 15. The ratio of extensional rate in the mix flow at various radius ratio.

...ity gradually increases with the increase of radius ratio in other positions. When $\kappa = 0.5$, the values for velocity are a concave function of $\delta^*$, while it is replaced by a convex function of the $\delta^*$ gradually with radius ratio increasing. Here, it is worth noting that the velocity is a nearly linear function of the $\delta^*$ when $\kappa = 0.6$. And it occurs before the changes of velocity, which is mainly because both the velocity of the melt and the volume flow rate will increase with the rise of radius ratio. Two noteworthy points near the center of the flow are extensional flow and the rapid increase of velocity. While the flow is pure shear flow near the inner and outer cylinders surface, melt velocity in this area does not increase. The radial distribution of the shear rate is depicted in Fig. 14b. In agreement with the former velocity analysis, when $\kappa < 0.7$, the shear rate along the radial direction gradually decreases and the minimum shear rate is obtained on the outer cylinder surface. When $\kappa > 0.7$, the shear rate along the radial direction gradually increases and the maximum shear strain rate is obtained on the outer cylinder surface. The shear rate is near equal to a constant along the radial direction when $\kappa = 0.7$.

According to the above analysis, the strength of the shear flow and extensional flow at the gap between the eccentric cylinders will change as the radius ratio changes. The ratio of extensional rates ($\phi$) at various radius ratios at the position of maximum extensional rate are shown in Fig. 15. We can find that the extensional rate decreases almost linearly with the increase of radius ratio. Though the extensional rate will increase with the increase of radius ratio, it is negligible compared with the change caused by the increase of shear rate. Therefore, as a whole, the ratio of extensional rate will decrease with the increase of radius ratio. In conclusion, the extensional rate and shear rate could be increased by increasing the radius ratio. And we can control not only the strength of extensional flow and shear flow but also the ratio of the extensional-shear flow by changing the radius ratio. To be specific, the ratio of shear flow in the mixture flow could be increased when increasing the radius ratio.

5. Conclusions

In this work the polymer flows between eccentric cylinders were investigated in a numerical way. The Bird-Carreau model was selected in order to describe the rheological behavior of the polymer melt flow between eccentric cylinders. The effects of the rotating speed, eccentricity, and radius ratio on the flow between eccentric cylinders were analyzed. The results indicated that we could adjust the strengths of shear flow and extensional flow by changing the rotating speed, eccentricity, and radius ratio. Specifically, the ratio of extensional flow in the mix flow would increase as the eccentricity increases, while the ratio of shear flow would increase when the radius ratio increases. Therefore, we can control the ratio of the shear-extensional flow by changing the eccentricity or the radius ratio.
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