Numerical study on turbulent blood flow in a stenosed artery bifurcation under periodic body acceleration using a modified $k$-$\varepsilon$ model

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Abstract

This article describes the numerical investigation of turbulent blood flow in the stenosed artery bifurcation under periodic acceleration of the human body. Numerical analyses for turbulent blood flow were performed for six simulation cases with different magnitude of periodic accelerations using a modified $k$-$\varepsilon$ turbulence model which is considering drag reduction of non-Newtonian fluid. The blood was considered to be a non-Newtonian fluid which is based on the power-law viscosity model. In order to validate the modified $k$-$\varepsilon$ model, numerical simulations were compared with laminar flow, the standard $k$-$\varepsilon$ model and the Malin’s turbulence model for power-law fluid. As results, laminar flow showed under predictions of blood velocity and wall shear stress, on the other hand, standard $k$-$\varepsilon$ model over estimates. The modified $k$-$\varepsilon$ model represents intermediate characteristics between laminar and standard $k$-$\varepsilon$ model, and the modified $k$-$\varepsilon$ model show good agreements with Malin’s verified power law model. Moreover, the computing time and computer resource of the modified $k$-$\varepsilon$ model are reduced about one third than low Reynolds number model including Malin’s model.

Keywords: turbulent blood flow, $k$-$\varepsilon$ turbulence model, periodic body acceleration, drag reduction, artery bifurcation

1. Introduction

The human body is flexible and deformable, particularly the softer tissues. However, when human body is imposed on a constant 16 G for a minute, may be deadly. To understand easily of magnitude of gravitational force, A high-performance automobile can brake at around 1 G. Although the human body is able to adapt to changes in the surroundings, a prolonged exposure of body to such vibrations leads to many health problems like headache, loss of vision, increase in pulse rate, abdominal pain, venous pooling of blood in the extremities and hemorrhage in the face, neck, eye sockets, lungs and brain (Hiatt et al., 1961; Hooks et al., 1972). Also when patients with orthostatic hypotension move from the lying-down position to the standing position, their blood pressures can decrease instantaneously by as much as 20 mmHg (Bradly and Davis, 2003), so these people easily feel vertigo and have temporary visual and hearing problems due to the rapid movement and vibration of the body.

Periodic acceleration (pGz) is forced from axial oscillating motion of whole body like a vibration. The human body often undergoes vibration and changes in gravitational forces in the daily activities of life, such as driving in vehicles, riding a roller coaster, and the rapid body movements that occur in sports activities. Due to physiological importance of body acceleration, many theoretical investigations have been carried out for the flow of blood under the influence of body acceleration and without stenosis. Sud and Sekhon (1985) presented a mathematical model for flow in single arteries subject to pulsatile pressure gradient as well as the body acceleration. The results shows that body acceleration does not affect the net flow of blood in arteries but affect the instantaneous value of flow variables such as flow rate, velocity and shear stress in the normal pumping actions. For body acceleration less than 0.01 G, the influence on blood flow in human arteries is negligible. However, when the body accelerations are more than about 0.5 G, the fluctuation of flow variables becomes relatively high. Since research of Sud and Sekhon (1985), various numbers of numerical studies (Sud and Sekhon, 1986; Chaturani and Palanisamy, 1990; Chaturani and Palanisamy, 1991; Chaturani et al., 1995) were performed and developed a mathematical model to analyze the effect of pGz on blood flow in healthy arteries.

A stenosis is an abnormal narrowing in a blood vessel and it disturbs the normal pattern of blood flow through the artery as reduced area. Moreover fluctuations of blood flow more increase by periodic acceleration. These flow
fluctuation and turbulence have bad effect to the blood vessel which make to accelerating the progress of stenosis. Chakravarty and Mandal (1996) studied the effect of body acceleration on pulsatile flow through stenosed arteries. Recently, El-Shahed (2003) showed the effects of externally imposed periodic body acceleration on pulsatile flow of blood through a stenosed porous artery considering blood as a Newtonian fluid. Mandal et al. (2007) used finite difference methods to develop a two-dimensional mathematical model for use in simulating stenosed arteries considering a moving wall effect with pre defined unsteady moving grid. Nagarani and Sarojamma (2008) investigated the effect of body acceleration on a mildly stenosed artery by modeling blood as a Casson fluid and Poiseuille flow using perturbation analysis assuming that the Womersley frequency parameter is small. Recently, Ro et al. (2009) performed three-dimensional analysis for blood flow characteristics with change of bifurcation angle and periodic acceleration in stenosed carotid artery. It showed that flow variables, such as flow rate and wall shear stress, increased with body acceleration and decreased with bifurcation angle. Also, high values of body acceleration generate back flow during the diastolic period, which increases flow fluctuation and the oscillatory shear index at the stenosis.

All previous research as above mentioned was carried out on the assumption that blood flow in stenosed artery is a laminar flow. In the healthy artery, blood flow in most arteries is in the laminar regime, but it is possible that flow could become turbulent at high values of body acceleration and severe stenosis (Ro and Ryoo, 2009; Glagov et al., 1988). Flow separation occurs after stenosis region at 70% stenosis due to a strong shear layer developed between the central jet and the recirculation region. For mild stenosis (below 25% stenosis), the wall and separation shear layers produce most of the losses. But severe stenosis (above 75% stenosis), turbulence is the major loss mechanism (Caro et al., 1996). Because the blood viscosity is shown the behavior of non-Newtonian fluid similar with polymer solutions as representative non-Newtonian fluid, a numerical simulation of turbulent flow in blood vessel is differ from Newtonian turbulent flow. A non-Newtonian fluid reduces the skin friction drastically in a turbulent flow due to phenomenon of drag reduction (Toms, 1976). The addition of flow concentration of polymers might be capable of improving blood flow through stenotic vessels without altering flow through normal vessels, as is suggested by Unthank et al. (1992). A non-Newtonian fluid has been researched to explain the mechanism of drag reduction. Moreover, several phenomenological models have been proposed to describe the turbulent fluids of drag-reducing fluids. Comprehensive reviews of the subject are provided in Hoyt (1972), Lumley (1973) and Virk (1975). In one of the early study attempts to analyze this phenomenon, Dodge and Metzer (1959) developed a correlation between friction factor and Reynolds number for turbulent pipe flow of purely viscous shear-thinning liquids based on a power-law representation of rheology. More recently, Pinho (2003) proposed a low-Reynolds number version of the turbulent flow model for drag reducing fluids. However, this model cannot determine model constants. The general turbulence model for a non-Newtonian fluid was developed by Malin (1997, 1999). Malin proposed a general low-Reynolds number k–ε turbulence model for power-law and Herschel-Bulkley fluids with an experimental correlation. However, low Reynolds number k–ε turbulence models including the Malin’s model have a weakness or limitations of grid generation at near wall region (y+≤ 5) with consideration of the viscous sub-layer than the standard k–ε turbulence model. Ro et al. (2010) proposed the modified k–ε turbulence model based on standard k–ε turbulence model by using drag reduction to analyze the turbulent flows of non-Newtonian fluids. This model shows good agreement with the experimental results. Also computation time and computer resource of the modified k–ε turbulence model are reduced by about one third than the low Reynolds number model for non-Newtonian fluid.

The numerical simulations for laminar blood flow in stenosed artery bifurcation with various percent stenosis, eccentricity of stenosis, bifurcation angle and magnitude of periodic acceleration are already carried out in our previous reaches (Ro and Ryoo, 2009). When 5.0 and 10.0 m/s² of periodic acceleration subjected in 75% stenosed vessel, at stenosis region, velocity and peak Reynolds number rapidly increase and Reynolds number reach transition and turbulent regime.

Therefore, in the present study, turbulent blood flow simulations using the modified k–ε turbulence model (Ro and Ryoo, 2010) are performed in the artery bifurcation with 75% stenosis under high value of periodic acceleration. For the validation of the result, it is compared with result of Malin’s general low-Reynolds number k–ε turbulence model (Malin, 1997) for power-law and standard k–ε turbulence model for Newtonian fluid.

2. Numerical Details

In order to simulate the characteristics of turbulent blood flow for various amplitudes of body acceleration of the idealized carotid artery with stenosis, three dimensional analyses are performed for six cases and simulation conditions of each case are listed in Table 1.

2.1. Modeling of a blood vessel

Fig. 1 shows a schematic view of the blood vessel, and it consists of bifurcation (α) angles (26°), percent stenosis (75%), and eccentricity (0%). Tang’s model (Tang et al., 2003) model is used to model the radius of stenosis.
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Let the radius of the symmetric stenotic vessel wall be given by

\[ H(z) = R_0 - S_0 \left[ 1 - \cos \left( 2\pi \frac{z}{z_1} \right) \right] \left( \frac{z_1 - z}{z_2 - z_1} \right), \quad z_1 \leq z \leq z_2 \]  

(1)

The asymmetric stenosis geometry is obtained by moving the centerline of the symmetric stenotic vessel wall.

\[ d(z) = d_{\text{sym}} \left[ 1 - \cos \left( 2\pi \frac{z}{z_1} \right) \right] \left( \frac{z_1 - z}{z_2 - z_1} \right), \quad z_1 \leq z \leq z_2 \]  

(2)

where \( d_{\text{sym}} \) is the distance moved by the centerline at the throat, \( R_0 \) is the radius of the blood vessel without stenosis (4 mm), \( z_1 \) (72 mm) and \( z_2 \) (88 mm) are the length of beginning and ending of the stenosis, the percent stenosis \( (S_0) \) and eccentricity \( (ECC) \) defined as

\[ S_0 = \frac{R_0 - R_{\text{throat}}}{R_0} \times 100\% \]  

(3)

\[ ECC = d_{\text{sym}} / (R_0 - S_0) \times 100\% \]  

(4)

2.2 Governing equation and boundary condition

Periodic body acceleration is represented using sinusoidal functions that are composed of the frequency and the magnitude of acceleration pulsation. (Sud and Sekhon, 1986)

\[ a(t) = a_0 \cos (\omega_b t + \phi) \]  

(5)

where \( a_0 \) is an magnitude of acceleration (5.0 and 10.0 m/s²), \( \omega_b \) is the frequency of periodic acceleration \( (2\pi f_p, f_p = 1.2 \text{ Hz}) \), \( \phi \) is the phase angle \( (0^\circ) \).

For the pulsatile flow of blood, a pressure difference at the inlet and outlet boundaries are applied as a sinusoidal pressure pulse (Mandal et al., 2007).

\[ \Delta P(t) = (A_0 + A \cos(\phi, t)) \]  

(6)

where \( A_0 \) is the constant component (steady-state part) of the pressure gradient as considering blood flowrate in the carotid artery (1000 pa), \( A_1 \) is the amplitude of its oscillatory component (200 pa), and \( \omega_p \) is the frequency of heart pulsation. Pressure pulse has the same frequency as the periodic acceleration \( (2\pi f_p, f_p = 1.2 \text{ Hz}) \), and the outlet boundary condition is applied as a steady-state pressure boundary condition (Mandal et al. 2007). Pressure difference and periodic acceleration profile are represented in Fig. 2.

The blood vessel wall is considered to be a rigid wall, and a no-slip condition is applied to the wall. In order to simulate a bifurcated blood vessel, three-dimensional flow analysis is performed.

The governing equations include the continuity equation and the momentum conservation equation, as follows:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \]  

(7)

Table 1. Simulation conditions for turbulent blood flow in severe stenosed artery bifurcation

<table>
<thead>
<tr>
<th>Case</th>
<th>Percent stenosis (S₀)</th>
<th>PGZ (a₀)</th>
<th>Turbulence model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>75%</td>
<td>5.0 m/s²</td>
<td>Laminar</td>
</tr>
<tr>
<td>2</td>
<td>75%</td>
<td>5.0 m/s²</td>
<td>Standard k-ε</td>
</tr>
<tr>
<td>3</td>
<td>75%</td>
<td>10.0 m/s²</td>
<td>Modified k-ε</td>
</tr>
<tr>
<td>4</td>
<td>75%</td>
<td>10.0 m/s²</td>
<td>Standard k-ε</td>
</tr>
<tr>
<td>5</td>
<td>75%</td>
<td>10.0 m/s²</td>
<td>Modified k-ε</td>
</tr>
<tr>
<td>6</td>
<td>75%</td>
<td>10.0 m/s²</td>
<td>Malin LRN k-ε</td>
</tr>
</tbody>
</table>

Fig. 1. Schematic representation of artery bifurcation for the turbulent blood flow under periodic acceleration.
The term $\gamma$ indicates the shear rate, represented as follows:

$$\gamma = \frac{1}{\rho} \sum_j \frac{\partial \vec{u}_j}{\partial t}$$  \hspace{1cm} (10)

The cross, power, and Carreau models are the representative constitutive equations of non-Newtonian fluid viscosity. In this study, the modified power-law model (Yoo et al., 1996) is used for the blood rheology.

$$\eta_0 < \eta = K \gamma^n < \eta_\infty$$  \hspace{1cm} (11)

where $\eta_0$ is the zero shear viscosity (0.056 Pa s), $\eta_\infty$ is the infinite shear viscosity (0.00345 Pa s), $K$ is the consistency index of power-law model (0.027 s), and $n$ is the power law index (0.53). The blood viscosity is shown in Fig. 3 with experimental data and the Carreau viscosity model (Biro, 1982).

2.3. Turbulence model for non-Newtonian fluid

2.3.1. Malin’s low Reynolds turbulence model for power law fluid

The paper of Malin (1997) reports on the numerical computation of the turbulent flow of power-law fluids in smooth circular tubes. A power-law fluid relates shear stress to strain rate via the consistency index $K$ and the power-law index $n$. The turbulence model for a power-law fluid is represented by a modified version of an existing low Reynolds number turbulence model (LB model).

The general form of the kinetic energy, dissipation rate and eddy viscosity in a low Reynolds number $k$–$\varepsilon$ model are as follows:

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \nu \left( \frac{\partial^2 U_i}{\partial x_i \partial x_j} + \frac{\partial^2 U_j}{\partial x_i \partial x_j} \right) \frac{\partial^2 \rho}{\partial x_j} \left[ \nu + \nu_n \right] \frac{\partial^2 \rho}{\partial x_i^2} + D$$ \hspace{1cm} (12)

$$\frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_j} = f \left( \frac{\partial U_i}{\partial x_i} \frac{\partial^2 \rho}{\partial x_j^2} + \frac{\partial U_j}{\partial x_i} \frac{\partial^2 \rho}{\partial x_i \partial x_j} - \frac{\partial^2 \rho}{\partial x_j^2} \right) \frac{\partial^2 \rho}{\partial x_i^2} + E$$ \hspace{1cm} (13)

$$\nu = f \nu_n \frac{k^2}{\varepsilon}$$ \hspace{1cm} (14)

The damping function of the Malin (1997) turbulence model for a power-law fluid is presented by the following equation:

$$f_n = \left[ 1 - \exp(-0.00165 \text{Re}_c/n') \right] (1 + 20.5 \cdot \text{Re}_c)$$ \hspace{1cm} (15)

The damping function $f_n$ of the Malin’s model differs from that of the LB model, and it includes the empirical correlation factor $n'$ , which considers the characteristics of a non-Newtonian fluid. This correlation factor in the Malin’s turbulence model increases the damping effect. The viscosity of a shear thinning fluid decreases at near wall due
2.3.2. Modified $k$–$\varepsilon$ turbulence model for power-law fluid

The general turbulence model for all types of non-Newtonian fluid does not proposed because the mechanism of drag reduction phenomenon is still indefinite. But according to the experimental study (Escudier et al., 1999; Ptasinski et al., 2001), non-Newtonian fluids have specific value of drag reduction in accordance with the Reynold's numbers and viscosity. The standard $k$–$\varepsilon$ turbulence model can consider drag reduction, it will be able to analyze the turbulent flow of a non-Newtonian fluid. This is the main idea of the modified turbulence model and modified $k$–$\varepsilon$ turbulence model can effectively predict the behavior of highly shear thinning fluid using a wall function.

From the experiment of Escudier et al. (1999), the mean velocity profile of a non-Newtonian turbulent flow with the phenomenon of drag reduction is no different from that of a Newtonian turbulent flow, in the standard $u^+–y^+$ form within the viscous sublayer region where $y^+$ is less than 10. This means that the viscous sublayer region does not play an important role in the drag reduction process, whereas the buffer layer region plays a major role in the drag reduction process. As many previous studies have shown, the buffer region increases in thickness with increasing levels of drag reduction, and the velocity distribution approaches Virk’s asymptote (1975).

In non-Newtonian turbulent flow, because the slope of the mean velocity profile is varied by drag reduction, turbulent viscosity changes according to the variation of drag reduction. Therefore, turbulent viscosity of non-Newtonian turbulent flow is represented as follows

$$v_t = F_s \nu_F \frac{k^2}{\varepsilon}$$  \hspace{1cm} (16)

where, $F_s$ is the damping function that varies according to the drag reduction. When drag reduction does not exist, such as in the turbulent flow of a Newtonian fluid, $F_s$ becomes unity. Therefore, the damping function $F_s$ is represented as follows

$$F_s = (1.0 - A \times Dr \times B)^2$$  \hspace{1cm} (17)

where, $A$ and $B$ are correlation factors. $Dr$ is the degree of drag reduction and it is defined as follows

$$Dr = \frac{Dr\%}{Dr\%_{\text{max}}}$$  \hspace{1cm} (18)

$Dr\%$ represents the quantity of drag, $Dr\%_{\text{max}}$ are calculated by the maximum drag reduction theory of Virk (1975) as follows

$$Dr\%_{\text{max}} = \frac{f - f_N}{f_N} \times 100\%$$  \hspace{1cm} (19)

$$Dr\%_{\text{max}} = \frac{f - f_{\text{Lin}}}{f_N} \times 100\%$$  \hspace{1cm} (20)

where, $f$ is friction factor. Subscription $N$, $NN$ are meaning of Newtonian and non-Newtonian fluid.

In the damping function of equation (17), correlation factor $A$ is a primary function of the friction factor, and correlation factor $B$ play a role in the change of the turbulent mean velocity profile. Correlation factors are defined with the experimental data of Dodge and Metzner (1959) and Shenoy et al. (1966).

$$A = \frac{1}{1.16 + 4.36e^{-5.13n^2} + (6.48e^{-2.29e^{-0.5(1.58e^{-n^2})}})Re_x}$$  \hspace{1cm} (21)

$$B = \exp((-0.015 \times n^{-0.25}) \times y')$$  \hspace{1cm} (22)

Eqs. (21) and (22) are composed by the general coefficients (power-law index $n$ and Reynolds number) of a power-law fluid, and therefore, the modified $k$–$\varepsilon$ turbulence model has a generality for shear thinning non-Newtonian fluids.

2.4. CFD Methods and grid generation

Numerical simulations are performed using FLUENT V6.3 with User-Defined Function (UDF) for the viscosity model, turbulence model, pressure and periodic acceleration profiles. UDF are applied by using shear rate function in viscosity model, and pressure and periodic acceleration profile are coded with suicidal function of time variation in inlet boundary condition and source term of momentum equation.

The commercial CFD software FLUENT is based on the finite volume method and the second order scheme is used for the discretization of the governing equation. Convergence criteria for numerical results are residuals ($10^{-5}$) of mass, velocity and turbulence quantity for all cells, and integrated quantities such as volumetric flow rate, inlet velocity and pressure are also examined. Unsteady flow analysis is conducted with a time step of 2 milliseconds and 4 periods.

Fig. 4 shows grid generation for stenosed artery bifurcation. The computational grid consisted of hexahedral meshes for three-dimensional geometry, and the commercial grid-generation software, ICEM-CFD, is used. Grid independence is achieved with several refinements of the grid for laminar, standard and modified $k$–$\varepsilon$ model which are used about number of 180,000 cells ($20 \times 20 \times 450$), 281,250 cells ($25 \times 25 \times 450$), and 405,000 cells ($30 \times 30 \times 450$). The difference of averaged axial velocity is within 5% for a period of pulsatile between medium size mesh and small mesh size. Thus, We chose a grid size of approx-
approximately 0.45 mm with 281,250 cells (25 x 25 x 450) and used refined grids of 0.2 mm on the near wall for laminar, standard and modified k-ε model. In order to compare with Malin’s non-Newtonian turbulence model, grid of Malin’s model is also composed about 800,000 cells in the same way.

3. Results and Discussion

3.1 The flow characteristics

The results of volumetric flow rate at the outlet of the branched artery are displayed in Fig. 5. The volumetric flow rate is shown as a periodic flow pattern after two periods because of the initial condition. The flow rate changed in proportion to the magnitude of body acceleration due to the same frequency of heart pulse and periodic body acceleration as shown in Fig. 2. However, there is almost no change in the net flow rate over complete pulse cycles. Periodic body acceleration affected the instantaneous flow rate in the pulse cycle. The instantaneous flow rate became a negative value when subjected high value of periodic acceleration and massive backflow is observed during the diastolic period. Volumetric flow rate increase by pressure force and periodic acceleration at the systolic period, on the other hand only periodic acceleration is applied at the diastolic period. For this reason, periodic acceleration make a higher blood flow rate and backflow rate in a cyclic period and backflow rate highly depends on magnitude of acceleration. These phenomena by periodic acceleration also are found on the study of El-Shahed (2003), Mandal et al. (2007) and Sud and Sekhon (1986).

Fig. 4. Grid generation for stenosed artery bifurcation. (a) Top view, (b) standard and modified k-ε and (c) Malin LRN k-ε.

Fig. 5. Volumetric flow rate at the outlet with magnitude of periodic acceleration.

Fig. 6. Axial velocity distributions in the middle of stenosis with magnitude of acceleration in maximum systole. (a) $a_0=5.0 \text{ m/s}^2$ (Case 1-3), (b) $a_0=10.0 \text{ m/s}^2$ (Case 4-6).
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Fig. 6 and 7 illustrates an axial velocity profile along the radial direction at the middle of stenosed region. The shape of the axial velocity in the core region is not a parabolic curve due to the shear-thinning characteristics of the blood. It can be seen that the axial velocity distribution is similar to the results of flow rate shown in Fig. 5. Axial velocity increased as body acceleration increased during the systolic period, and back flow occurred with a periodic acceleration during the diastolic period. Considering infinite shear viscosity, the Reynolds number is about 3,900 and 4,900 under 5.0 and 10.0 m/s$^2$ of acceleration magnitude. A Reynolds number between 2100 and 4000 is known as transitional flow. In cases of 5.0 m/s$^2$ of acceleration magnitude, blood flow is transitional flow or weakly turbulent flow, so laminar flow simulations are also performed in this case. Axial velocity of core region using the standard $k$-$\varepsilon$ model is, on the whole, over predicted than results from laminar and the modified $k$-$\varepsilon$ model simulation. On the other hand, axial velocity at near wall is under-predicted than others. Newtonian turbulence models including standard $k$-$\varepsilon$ model do not consider drag reduction of non-Newtonian fluid at the wall. If Newtonian turbulence model is used for non-Newtonian turbulent flow, wall velocity decrease by high friction factor and core velocity increase by momentum conservation. Prediction results have more differences in the maximum systole due to increase of drag reduction effect but almost same velocity profile is shown in maximum diastole due to small drag reduction as lower blood velocity. Because the modified $k$-$\varepsilon$ model is developed considering drag reduction, it shows good agreements with Malin’s non-Newtonian turbulence model.
The axial velocity distributions in the middle of the branched artery are shown in Fig. 8 and 9. The axial velocity at near inner wall is greater than the outer wall as increase of periodic acceleration in maximum systole because inertial force is higher than centrifugal force caused by the geometrical shape of bifurcation. As a reason, appearance of local flow concentration did not occur during the diastolic period.

In cases of laminar flow, velocity difference between peak and mean velocity is higher than results of turbulent flow simulations. Viscous effect of laminar flow is higher than turbulent flow and turbulence is quickly diffused to the flow field. It plays a role in flow stabilization. The tendencies and results of standard $k-\varepsilon$ model are completely disagree with modified $k-\varepsilon$ model and Malin's model. It also does not predict the negative and circulation flow at the branched artery owing to over estimation of turbulence. Negative velocity or circulating flow in the bifurcated artery is important that regions of circulating and stagnating flow are a major cause of arterial disease.

More detailed results of internal flow distributions are shown in Fig. 10 and 11.

In Fig. 10, velocity contours in the cross section of axial direction with $a_0 = 5.0 \text{ m/s}^2$ (Case 1~3) (a) Maximum systole, (b) Maximum diastole.

Fig. 10. Axial velocity contours in the cross section of axial direction with $a_0 = 5.0 \text{ m/s}^2$ (Case 1~3) (a) Maximum systole, (b) Maximum diastole.

Fig. 11. Axial velocity contours in the cross section of axial direction with $a_0 = 10.0 \text{ m/s}^2$ (Case 4~6) (a) Maximum systole, (b) Maximum diastole.
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...cosity of blood rheology. Therefore, modified $k-\varepsilon$ model shows a similar results to laminar flow, but value of axial velocity is lower than result of laminar simulation because of higher prediction of friction factor in the characteristics of turbulent flow simulation.

Fig. 11 shows a axial velocity contour of turbulent flow analysis. As a similar reasons with result of transition region, Malin’s and modified $k-\varepsilon$ model are show incredibly similar results with considering drag reduction phenomenon of shear thinning fluid but Standard $k-\varepsilon$ model also does not predict not only turbulent flow filed of shear thinning fluid but laminar flow.

3.2. The wall shear stress and oscillatory shear index

Fig. 12 and 13 represent maximum wall shear stress (WSS) at inner and outer wall in maximum systole. Generally, the WSS of a healthy aorta is about 1–2 Pa, but a stenosed artery may have a WSS of more than 30 Pa (Fry, 1968). The uniform WSS is shown in the wall of the upstream, and the two peak values of the WSS are indicated at the middle of stenosis and the apex of bifurcation due to high velocity by area reduction. Downstream of the branched artery, WSS at the inner and outer wall rapidly decrease due to redeveloping flows. Standard $k-\varepsilon$ model over predicted WSS as expected because WSS is related to the shear rate and the velocity gradient of a blood flow at the wall. The modified $k-\varepsilon$ model shows about 20% lower value of WSS than standard $k-\varepsilon$ model at outer wall owing to consideration of drag reduction. At inner wall, differences of WSS more than doubled between standard and modified $k-\varepsilon$ model. High value of periodic acceleration enormously increases WSS in the stenotic region and it could be dangerous in a patient with stenosis.

Fig. 12. Wall shear stress along outer wall and inner wall with $a_0=5.0 \text{ m/s}^2$ in maximum systole. (Case 1–3)

Fig. 13. Wall shear stress along outer wall and inner wall with $a_0=10.0 \text{ m/s}^2$ in maximum systole. (Case 4–6)
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Fry (1968) and Caro et al. (1996) postulated that higher WSS values could damage the arterial wall and endothelial disruption or denudation could lead to atherosclerosis. Also, recently it has become widely accepted that extreme low WSS or oscillatory shear stress, rather than high WSS value, contribute to atherosclerosis (Ku et al., 1985; Malek et al., 1999).

The oscillatory shear index (OSI) defined by Ku et al. (1985) is an important parameter for the prediction of arterial disease. The OSI is defined by the following equation:

\[
\text{OSI} = \frac{\frac{1}{T} \int_{0}^{T} \tau_{w} \, dt}{\frac{1}{T} \int_{0}^{T} \tau_{w}^* \, dt}
\]

where \(\tau_{w}\) is the instantaneous WSS vector, and \(\tau_{w}^*\) is the WSS component acting in the direction opposite to the mean WSS vector. The OSI is a dimensionless value and does not take account for the magnitude of the shear stress vectors. For purely oscillatory flow, the OSI reaches its maximum value of 0.5. As the shear stress acting in the opposite direction increases, the value of OSI increases, and the largest value of the OSI occurs in the flow separation and recirculation region.

Fig. 14 compares the OSI contour for each case. High values of OSI are shown at middle of stenosis and branched artery. It has been reported that atherosclerosis generally occurs at the branched artery due to the sudden change of section area and the curvature of the artery (Berger and Jou, 2000). When subjected high value of periodic acceleration, the value of OSI and flow fluctuation increase by massive back flow and instantaneous negative velocity in the branched artery. Due to flow concentration in branched artery, modified \(k-\varepsilon\) model and the Malin’s model show strong oscillatory shear flow in the branched artery.

As results from flow variables of turbulent blood flow, it is essential to accurate prediction of blood flow because other important quantities such as values of WSS and OSI are highly associated with flow variables. The standard \(k-\varepsilon\) model is not suited for non-Newtonian turbulent flow due to absence of considering damping effect at near wall and it shows mainly over predicted results. The modified \(k-\varepsilon\) model and Malin’s model have the characteristics intermediate between laminar and standard \(k-\varepsilon\) model. Validation of modified \(k-\varepsilon\) model is already conducted with experiments (Ro and Ryou, 2010) and it is expected to get the more acceptable results in turbulent flow of hemodynamics and power law fluid.

4. Summary and conclusions

The main goal of this paper is to investigate characteristics of turbulent blood flow using a modified \(k-\varepsilon\) model in the stenosed artery bifurcation under periodic acceleration.

In conclusion, periodic body acceleration affected instantaneous flow variables such as volumetric flow rate and blood velocity, and amplitudes of fluctuation increased as imposed body acceleration increased. When subjected high value of periodic acceleration, blood flow is changed turbulent flow. A friction factor and the slope of the mean velocity profile are varied according to the drag reduction in the non-Newtonian turbulent flow. Non-Newtonian turbulence model for blood flow is needed to consider the phenomenon of drag reduction. Therefore, the modified \(k-\varepsilon\) model considering drag reduction is proposed.

In order to validate the modified \(k-\varepsilon\) model, numerical simulations are compared with laminar flow, the standard \(k-\varepsilon\) model and the Malin’s turbulence model for power-law fluid. Laminar flow shows an under predictions of axial blood velocity, wall shear stress and OSI, on the other hand, standard \(k-\varepsilon\) model over estimates results of simulation. The modified \(k-\varepsilon\) model represents intermediate characteristics between laminar and standard \(k-\varepsilon\) model, and the modified \(k-\varepsilon\) model show good agreements with Malin’s verified power law model in the simulation of blood flow. Moreover, the computing time and computer resource of the modified \(k-\varepsilon\) model are reduced about one third than low Reynolds number model including Malin’s model.

So, the modified \(k-\varepsilon\) model is expected to get the more acceptable and economical results in turbulent blood flow.

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References


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