Optimization of Some Radiators With Fins and With Evolute Reflectors

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(Received March 13, 1965)

The maximum heat rejection per unit weight of radiator with evolute type reflectors is compared with those of some radiators with optimum rectangular fins by means of examples. The mutual irradiations among the surfaces of neighboring radiator parts are considered in computing the view factors.

The heat rejection system for space power plants is a major weight item. For this reason a number of papers in recent years treated the utilization of the some kind of fins or reflectors. Conducting fins that act as extended heat transfer surface are spaced between the tubes. If the cylindrical radiator geometry is selected, a radiator with evolute reflectors may be interesting. Some radiator configurations are shown in Fig. 1.

An approximate solution of maximizing heat rejection per unit weight with tube and fin geometry is given in Callinan and Berggren by assuming that the fin view factor equals one. It is also proposed in his work, that as a result of future analytical work, a generalized corrector factor could be generated, which could readjust the fin heat rejection rate, as calculated by the approximate method, to show closer agreement with the exact solution. Instead of finding a correction factor the fin and tube view factors are derived analytically only as a function of $2B/D$ in Appendices A and B. Using these view factors the heat rejection per unit weight is maximized with optimum rectangular fins.

Reynolds presented the optimization of the fin geometry in consideration of the incident irradiation and the associated structure weight. According to his paper the optimum fin height and thickness are expressed in terms of the heat $(Q/L)$, which must be rejected from each fin per unit of span. It means, one cannot determine the optimum fin height and thickness, until the radiator geometry including fin view factor and efficiency is known.

A method and examples of maximizing heat rejection per unit weight per unit radiator length are given and results for some possible radiator configurations are compared.
The following assumptions are made:
1. Material properties, $k, e$, are independent of the temperature.
2. The fin thickness is constant.
3. The fin base temperature is constant at $T_0$.
4. Incident solar radiation is excluded.
5. The increase of the associated structure weight according to the increase of the fin height is not considered.
6. The radiator is infinitely long in tube axial direction.
7. The surface heat transfer is entirely by radiation.

(1) A tube and Two Fins with Double-active Surfaces

Considering a section which includes half a tube and a fin in Fig. 1 (a), the heat rejected from the fin per unit length is

$$\langle Q/L \rangle_f = bT_0 e \epsilon T_0^4 B$$  \hspace{1cm} (1.1)

The fin view factor $F_f$ is calculated in Appendix $\Lambda$ and illustrated in Fig. 4.

$$F_f = \sqrt{1 - D/2B - D/2B \cos \left( \frac{D/2B}{2 + D/2B} \right)}$$  \hspace{1cm} (1.2)

If the irradiation from the tube section is neglected, $\langle Q/L \rangle_f$ may be expressed as in Hwang-Bo$^{(4)}$.

$$\langle Q/L \rangle_f = -\frac{1}{1.76} F_f \epsilon T_0 A^4 B$$  \hspace{1cm} (1.3)

The heat rejected from the tube section is

$$\langle Q/L \rangle_t = \left( \frac{\pi D}{2} - d \right) e \epsilon T_0 A^4 F_f$$  \hspace{1cm} (1.4)

The tube view factor $F_t$ is calculated in Appendix B and illustrated in Fig. 5.

$$F_t = \frac{1}{2} + \frac{2}{\pi} \left( 1 + \frac{2B}{D} - \sqrt{\left( \frac{2B}{D} \right)^2 + \frac{2B}{D}} \right. \hspace{1cm} (1.5)$$

$$- \frac{1}{2} \arcsin \left( \frac{0.5}{2B/D + 0.5} \right)$$

The heat rejection per unit weight of radiator may be described as a function of the optimum fin height.

$$\langle Q/L \cdot G \rangle_f = \frac{\pi D}{2} \left( \frac{\pi D}{2} - d \right) e \epsilon T_0 A^4 F_f + 2 \eta B_\text{sh} \epsilon \epsilon T_0 A^4 F_f$$

$$\left( \frac{D^2}{4} \left( \frac{D - 2d}{4} \right)^2 \right) \rho_1 + B_\text{sh} \cdot d \rho_2$$  \hspace{1cm} (1.6)

The optimum rectangular fin thickness $d$ is given in Hwang-Bo$^{(4)}$ in Fig. 3.

$$d_{\text{opt}} = \frac{5}{2} \sqrt{\frac{1.46 \epsilon \epsilon T_0 A^4 F_f}{k T_0 e \epsilon T_0 A^4 F_f + b \langle Q/L \rangle_f}}$$  \hspace{1cm} (1.7)

Ex. 1.

With the following assumed values

Two fins $N_f = 2$
Double-active surface $b = 2$
Tube diameter $D = 14$ [mm]
Tube wall thickness $s = 2$ [mm]
Thermal conductivity $k = 0.2$ [W/cm°C]
Fin base temperature $T_0 = 800$ [°C]
Emissivity of the tube surface $\epsilon_t = 0.9$
Emissivity of the fin surface $\epsilon_f = 0.85$
Density of the tube $\rho_1 = 7.9$ [g/cm³]
Density of the fin $\rho_2 = 4.3$ [g/cm³]

The maximum heat rejection is found at the optimum fin height $B_\text{sh} = 13.7$ [mm] from Fig. 2.

$$\langle Q/L \cdot G \rangle_{\text{max}} = 5.9$$ [W/cm·g]
The optimum fin thickness is from Fig. 3 $d_{\text{opt}} = 1.14$ [mm]

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Fig. 1. Schematic Radiator Configurations with Fins or Reflectors.
(2) A Tube and Four Fins with Double-active Surfaces

Considering a section which includes two half tubes and four fins in Fig. 1(c), the heat rejected from the section is

\[ \frac{Q}{L} = 4B \eta_{F2} F_2 (\varepsilon_r T_s^4 + \gamma_2 H_2) + 4B \eta_{F3} (\varepsilon_r T_s^4 + \gamma_3 H_3) + (\pi D - 4d) (\varepsilon_r T_s^4 + \gamma_3 H_3) F_3 \]  

(1.8)

Where \( H_1 \) is the irradiation leaving the adjacent radiator surface and may be given approximately as follows.

Neglecting the twice or more reflected radiation among the surfaces \( A_1 \), \( A_2 \), and \( A_3 \),

\[ H_1 = 8 \varepsilon_r T_s^4 B (\eta_{F1} + \eta_{F3} F_3) + (\pi D - 4d) \varepsilon_r T_s^4 F_3 \]  

(1.9.1)  

\[ H_2 = 4 \varepsilon_r T_s^4 B (\eta_{F1} + \eta_{F2} F_2) + (\pi D - 4d) \varepsilon_r T_s^4 F_2 \]  

(1.9.2)  

\[ H_3 = 4 \varepsilon_r T_s^4 B (\eta_{F1} + \eta_{F3} F_3) + (\pi D - d) \varepsilon_r T_s^4 F_3 \]  

(1.9.3)

The heat rejection per unit weight of radiator may be expressed as a function of the optimum fin height.

Both fin efficiencies are slightly different, since the irradiation from the adjacent radiator surfaces are not equal. A dimensionless parameter \( \alpha \), defined as \( H/\varepsilon_r T_s^4 F_3 \), is introduced to compute the fin efficiencies. With the view factors given in Appendix C and Fig. 5(a) in Reynolds \( \eta_1 = 0.61 \) and \( \eta_2 = 0.59 \) are found.

\[ \frac{Q}{L} = \frac{\pi D}{4} - \frac{(D - 2d)^2}{4} \eta_1 + 4Bd \frac{d}{2} \]  

(1.10)

With the same values assumed in Example 1, the optimum fin height is, from Fig. 2, \( B_{opt} = 2 \) [mm], and the optimum fin thickness is, from Fig. 3, \( d_{opt} = 0.45 \) [mm] with

\[ \frac{Q}{L} = 5.3 \text{ [W/cm•g]} \]

(3) A Tube and Two Fins with One Active surface

The heat rejection per unit weight can be written from equation (1.6)

\[ \frac{Q}{L} = \frac{\pi D}{4} - \frac{d}{2} \varepsilon_r T_s^4 F_1 + \eta_2 B_{opt} \varepsilon_r T_s^4 \]  

(1.11)

With the same conditions as in Example 1, the heat rejection is presented as a function of \( B_{opt} \) in Fig. 2.
\( \frac{Q}{L \cdot C_p} \) \text{max} = 2.7 [W/cm \cdot g] \\
at \text{B}_{\text{ref}} = 9.5 \text{ [mm]} \\
d_{\text{ref}} = 1 \text{ [mm]} \\

(4) \text{ A tube with Evolute Reflectors}

Reflective efficiency is defined as the ratio of the heat rejected by the tube with reflectors to that which would be rejected by the tube without reflectors with no incident irradiation.

For a very long radiator in axial direction the reflective efficiency is given in Hwang-Bo(5).

\[
\eta_{\text{ref}} = 0.196t^2 + 0.402t + 0.402 \tag{1, 12}
\]

With the following values assumed in Example 1,

\( T_x = 800 \text{ [°C]} \) \\
\( D = 14 \text{ [mm]} \) \\
\( \rho_1 = 7.9 \text{ [g/cm}^3] \) \\
\( \rho_3 = 8 \text{ [g/cm}^3] \)

and reflective reflectivity

\( \gamma = 0.94 \) \\
\( d_{\text{ref}} = 0.18 \text{ [mm]} \)

The integration of equation (A,1) results

\[
F_i = \int_0^{2\pi} \frac{1}{2B} \left( \sin \phi + \sin \phi' \right) dx \tag{A, 1}
\]

Where

\[
\sin \phi = \sqrt{x^2 + \pi D} \\
\sin \phi' = \sqrt{\left( \frac{D}{2} + 2B - x \right)^2 - \left( \frac{D}{2} \right)^2} \\
\]

The integration of equation (A,1) results

\[
F_i = \sqrt{1 + \frac{(D/2B)}{(2 + D/2B)}} \tag{A, 2}
\]

Appendix B. Tube Geometric Factor \( F_i \)

The tube view factor is equal to that fraction of the radiation, leaving the tube surface \( A_i \) in all direction, which is intercepted within the bounds of the imaginary surface \( A_i \) in Fig. 5. For the area of infinite extent in one direction, generated by a straight line moving always parallel to itself: all cross sections normal to the infinite dimension are identical, the view factor may be given as follows(2).

\[
F_i = \frac{A_3 + A_2 - (A_4 + A_6)}{2 \cdot A_3} \tag{A, 3}
\]

\[
F_i = \frac{\pi D}{4} - (2B + D) - \left[ \sqrt{\left( \frac{2B + D}{2} \right)^2 + \left( \frac{D}{2} \right)^2} \right] \tag{A, 4}
\]

Where

\[
\theta = \arcsin \left( \frac{D/2}{2B + D/2} \right)
\]

\[
F_i = 0.5 + \frac{2}{\pi} \left[ 1 + \left( \frac{2B}{D} \right) - \sqrt{\left( \frac{2B}{D} \right)^2 + \left( \frac{2B}{D} \right)} \right] - \frac{1}{2} \arcsin \left( \frac{0.5}{2B + 0.5} \right) \tag{A, 5}
\]

Appendix C. View Factors between Two Surfaces of \( A_1, A_2, A_3, \) and \( A_4 \)

The view factor \( F_{12} \) is equal to that fraction of the radiation leaving the surface \( A_4 \) in all directions, which is intercepted by the surface \( A_3 \) in Fig. 6. The method in Appendix B is still valid for all other view factors and the results are given in terms of \( 2B/D \) in Fig. 6.
\[ F_{13} = \sqrt{\frac{5}{4} \left( \frac{D}{2B} \right)^2 + \frac{1}{4} \left( \frac{D}{2B} \right)^2 - \frac{1}{2} \left( \frac{D}{2B} \right)^2} + \frac{1}{4} \]
\[ - \frac{1}{2} \left( \frac{D}{2B} \right) \arcsin \frac{D/2B}{1+D/2B} \quad (A, 6) \]

\[ F_{12} = \frac{2}{\pi} \left[ 1 + \frac{D}{B} + \sqrt{5 \left( \frac{B}{D} \right)^2 + \frac{1}{2}} \right] \quad \text{(A, 7)} \]
oder \[ F_{12} = \frac{2}{\pi} \left[ \frac{1}{2} \arccos \frac{1}{2B/D + 1} + \frac{B}{D} - \sqrt{\left( \frac{B}{D} \right)^2 + \frac{1}{4}} \right] \]

\[ F_{11} = \left[ 2 \sqrt{\left( \frac{B}{D} \right)^2 + \frac{1}{2}} \arcsin \frac{1}{2B/D + 0.5} \right] \quad \text{(A, 12)} \]

\[ F_{23} = 2.5 + \frac{D}{B} - \sqrt{4.25 + \frac{4D}{B} + \left( \frac{D}{B} \right)^2} \quad (A, 9) \]

**Literature**

(1) J. P. Callinan and W. P. Berggren,
Some Radiator Design Criteria for Space Vehicle.

(2) H. Hottel,

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**Fig. 4. Fin View Factor vs \( \frac{D}{2B} \)**

**Fig. 5. Tube Geometric Factor vs \( \frac{2B}{D} \)**

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Fig. 6. Fin and Tube View Factors vs $\frac{D}{2B}$